



UNIVERSITY OF LJUBLJANA

Faculty of Civil and Geodetic Engineering

Example of the analysis and design of a bridge according to EC8/2

T. Isaković

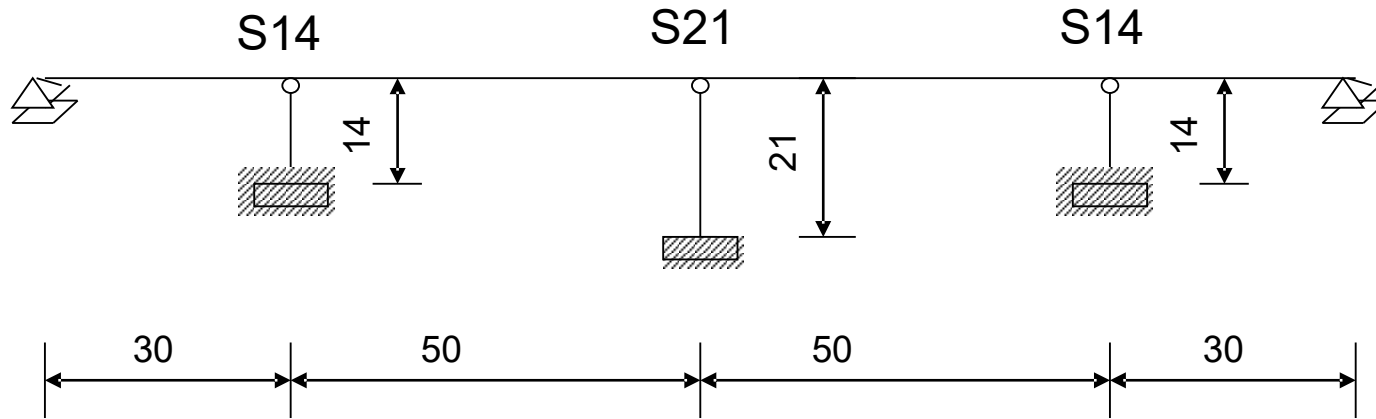


$a_{gR} = 0,25g$

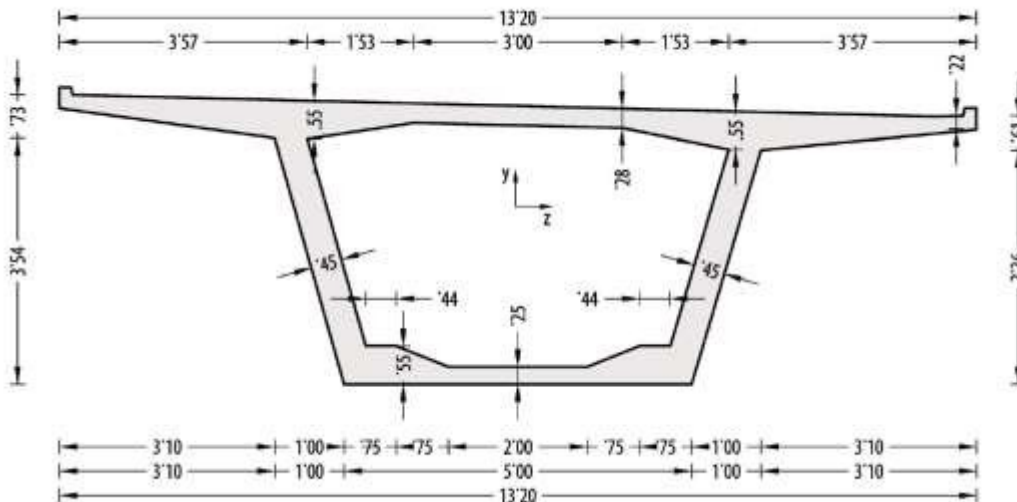
Soil C

Bridge II. importance class

Ductile response



Deck cross-section



Concrete C35/45 $E = 34 \text{ GPa}$

Steel B500

$A = 9,65 \text{ m}^2$

$I_t = 36,5 \text{ m}^4$

$A_{sy} = 3,82 \text{ m}^2$

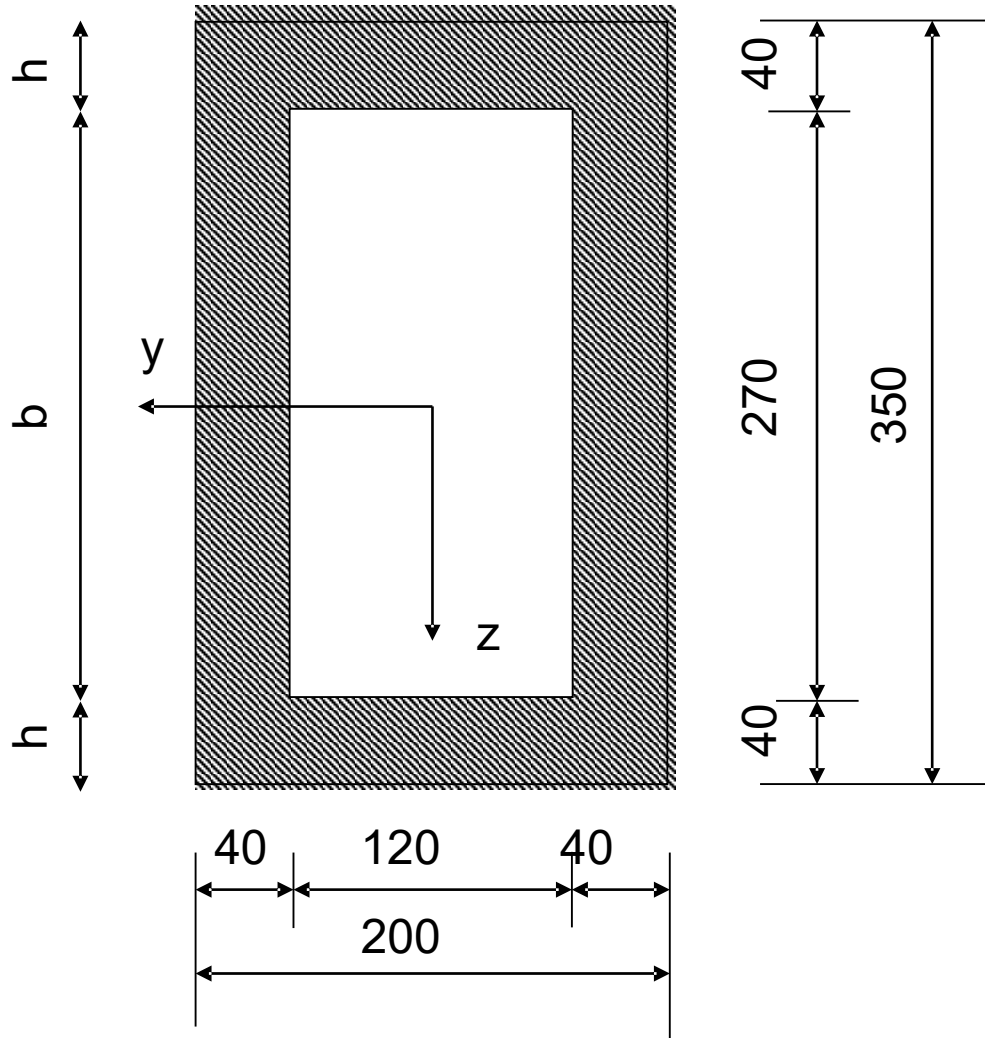
$I_y = 96 \text{ m}^4$

$A_{sz} = 6,93 \text{ m}^2$

$I_z = 21,5 \text{ m}^4$

According to EC8/2 It is reduced to $I_t = 36,5/2 = 18,25 \text{ m}^4$

Columns



$$C\ 25/30\ E = 3,1 \cdot 10^7\ \text{kN/m}^2$$

$$A = 3,76\ \text{m}^2$$

$$A_{sy} = 1,6\ \text{m}^2$$

$$A_{sz} = 2,8\ \text{m}^2$$

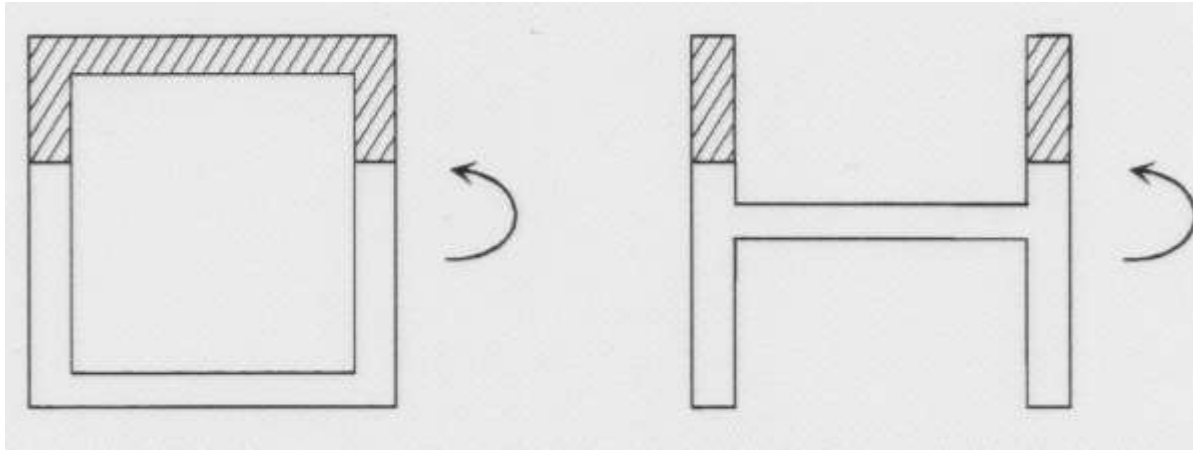
$$I_t = 4,19\ \text{m}^4$$

$$I_y = 5,18\ \text{m}^4$$

$$I_z = 1,94\ \text{m}^4$$

$$\frac{b}{h} = \frac{270}{40} = 6,75 < 8$$

- Box cross-section is favourable due to the large width of the compression zone, providing larger ductility

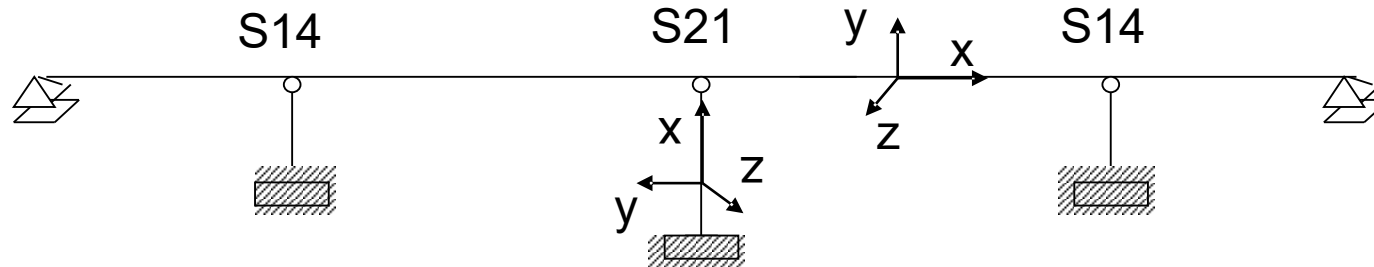


favourable

unfavourable

Analysis

Coordinate systems



Actions

Dead load - superstructure $g = 295 \text{ kN/m}$

Weight of piers $g_s = 94 \text{ kN/m}$

Masses

Bridge of II. importance class – only dead load is taken into account

Mass - deck $M_p = g / 9,81 = 295 / 9,81 = 30 \text{ t/m}$

Mass - piers $M_s = g_s / 9,81 = 94 / 9,81 = 9,6 \text{ t/m}$

Dead load

The axial forces occur in piers

Axial forces at the base of piers (weight of the piers is included)

$$N_{S14} = 14329 \text{ kN}$$

Normalized axial force

$$\eta = \frac{N_{Ed}}{A_c f_{ck}} = \frac{14329}{3,76 \cdot 25 \cdot 1000} = 0,152 < 0,2$$

$$N_{S21} = 17485 \text{ kN}$$

Normalized axial force

$$\eta = \frac{N_{Ed}}{A_c f_{ck}} = \frac{17485}{3,76 \cdot 25 \cdot 1000} = 0,186 < 0,2$$

Axial forces in piers are calculated using programme SAP2000

Seismic analysis in the longitudinal direction

Fundamental mode method – Rigid deck model

Field of application:

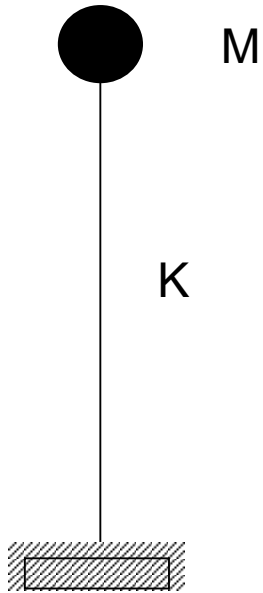
a) Mass of piers less than 20% of the mass of the deck

$$M_s = (14 + 21 + 14) \cdot 9,6 = 470 \text{ t}$$

$$M_p = 30 \cdot 160 = 4800 \text{ t} \quad M_s / M_p = 0,098$$

b) Eccentricity – Distance between the centre of mass and centre of stiffness is 0, the bridge is symmetric.

SDOF model of the bridge



$$T = 2 \pi \sqrt{\frac{M}{K}} = 2 \pi \sqrt{\frac{5035}{144998}} = 1,171s$$

Total mass of the structure

$$M = 30 \cdot 160 + (7 + 10,5 + 7) 9,6 = 5035 \text{ t}$$

Half of the piers' mass is added

Flexibility of piers

$$f = \frac{h^3}{3EI_z} + \frac{h}{GA_{sy}}$$

$$f_{S14} = \frac{14^3}{3 \cdot 3,1 \cdot 10^7 1,94} + \frac{14}{1,29 \cdot 10^7 1,6} = 1,589 \cdot 10^{-5}$$

$$f_{S21} = \frac{21^3}{3 \cdot 3,1 \cdot 10^7 1,94} + \frac{21}{1,29 \cdot 10^7 1,6} = 5,23 \cdot 10^{-5}$$

Stiffness of piers

$$k = 1/f$$

$$k_{S14} = 62947 \text{ kN/m}$$

$$k_{S21} = 19104 \text{ kN/m}$$

Stiffness of the structure

$$K = 2 \cdot k_{S14} + k_{S21} = 144998 \text{ kN/m}$$

Design acceleration spectrum

$$a_g = \gamma_I a_{gR} = 1 \cdot 0,25 \text{ g} = 0,25 \text{ g}$$

Soil C:

$$S = 1,15$$

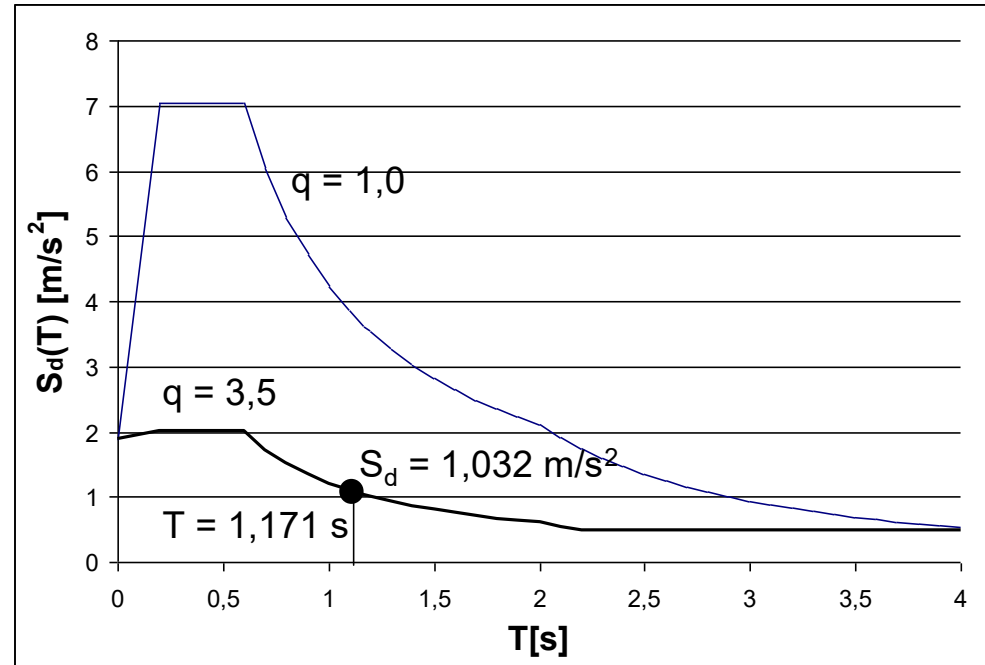
$$T_B = 0,2 \text{ s}$$

$$T_C = 0,6 \text{ s}$$

$$T_D = 2,0 \text{ s}$$

damping 5%

$$T_C = 0,6 < T = 1,171 \text{ s} < T_D = 2 \text{ s}$$



Shear span ratio of pier S14 $\alpha = 14/3,5 = 4 > 3 \Rightarrow q = 3,5$

$$S_d(T) = a_g S \frac{2,5}{q} \left(\frac{T_C}{T} \right) = 0,25 \cdot 9,81 \cdot 1,15 \frac{2,5}{3,5} \left(\frac{0,6}{1,171} \right) = 1,032 \frac{\text{m}}{\text{s}^2} > 0,2 \cdot 0,25 \cdot 9,81 = 0,49 \frac{\text{m}}{\text{s}^2}$$

Seismic force in the longitudinal direction

$$F = M S_d = 5035 \cdot 1,032 = 5196 \text{ kN}$$

Shear forces in piers

$$F_{s14} = \frac{k_{s14}}{k} F = \frac{62947}{144998} 5196 = 2256 \text{ kN}$$

$$F_{s21} = \frac{k_{s21}}{k} F = \frac{19104}{144998} 5196 = 685 \text{ kN}$$

Bending moments in piers

$$M_{s14} = F_{s14} h_{s14} = 2256 \cdot 14 = 31584 \text{ kNm}$$

$$M_{s21} = F_{s21} h_{s21} = 685 \cdot 21 = 14385 \text{ kNm}$$

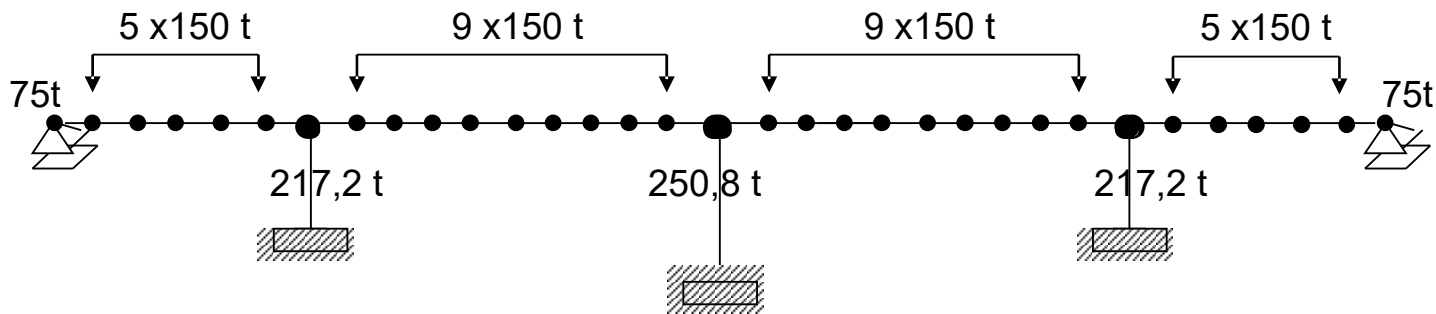
Seismic analysis in the transverse direction

Fundamental mode method – Flexible deck model (FMM)

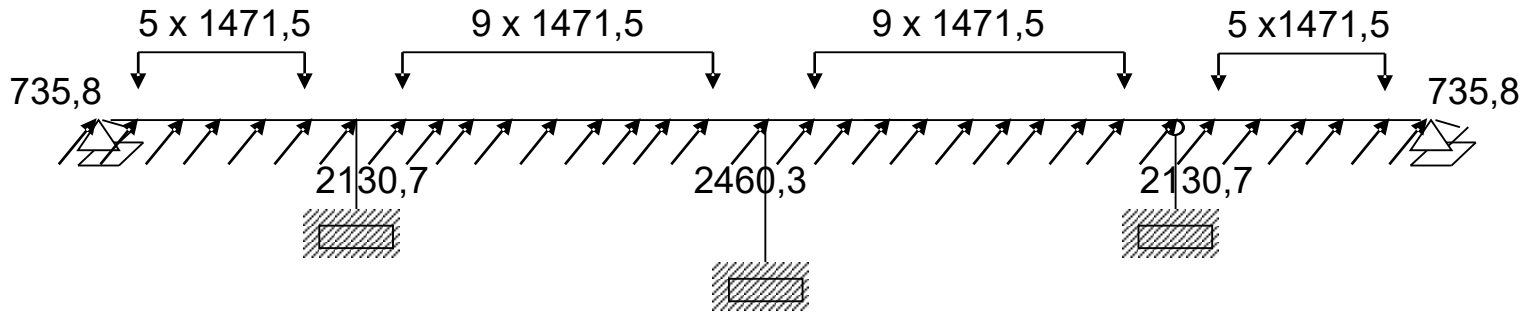
Structure is subjected to forces $F_i = M_i \cdot g$

Masses are concentrated in nodes at the equidistant lengths of 5m

Half of the mass of piers is added at relevant nodes



The scheme of the inertial forces $F_{ig} = M_i g$



Displacements d_i , corresponding to inertial forces F_{ig}

Table 1

node	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
d_i [m]	0,125	0,125	0,125	0,125	0,125	0,126	0,127	0,13	0,134	0,137	0,141	0,144	0,147	0,150	0,151	0,152	0,152
node	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	
d_i [m]	0,152	0,151	0,150	0,147	0,144	0,141	0,137	0,134	0,13	0,127	0,126	0,125	0,125	0,125	0,125	0,125	

average displacement is 0,135m, maximum difference is 0,0270 m

Ratio $0,027/0,135 = 0,20$ – the rigid deck model can be used, however this is the maximum allowed value, therefore the analysis is continued with flexible model

Using data in Table 1 the following data are obtained:

Period of vibration

$$T = 2 \pi \sqrt{\frac{\sum M_i d_i^2}{g \sum M_i d_i}} = 0,742s$$

Design acceleration

$$S_d(T) = a_g S \frac{2,5}{q} \left(\frac{T_c}{T} \right) = 0,25 \cdot 9,81 \cdot 1,15 \frac{2,5}{3,5} \left(\frac{0,6}{0,742} \right) = 1,629 \frac{m}{s^2}$$

Inertial forces F_i

$$F_i = \frac{4 \pi^2}{g T^2} S_d(T) d_i M_i = \frac{4 \pi^2}{9,81 \cdot 0,742^2} 1,629 d_i M_i = 11,90 \cdot d_i M_i$$

Table 2

node	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
F_i [kN]	111	223	223	223	223	225	328	232	239	244	251	257	262	267	269	271	453
node	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	
F_i [kN]	271	269	267	262	257	251	244	239	232	328	225	223	223	223	223	111	

Using programme SAP 2000 the following internal forces in piers are calculated

Bending moments

$$M_{s14} = 47710 \text{ kNm}$$

$$M_{s21} = 28247 \text{ kNm}$$

Shear forces

$$V_{s14} = 3408 \text{ kN}$$

$$V_{s21} = 1345 \text{ kN}$$

In some cases (e.g. in bridges supported by very short columns located near the centre of the bridge) the method can give unrealistic results

Thus an additional control, presented on the next slide, is performed

Displacements d_i (Table 1), corresponding to inertial forces $F_{ig} = M_i g$ are divided by the ratio $S_d(T)$ and g

$$S_d(T)/g = 1,629 / 9,81 = 0,166$$

Table 3 displacements $d_{i,0}$

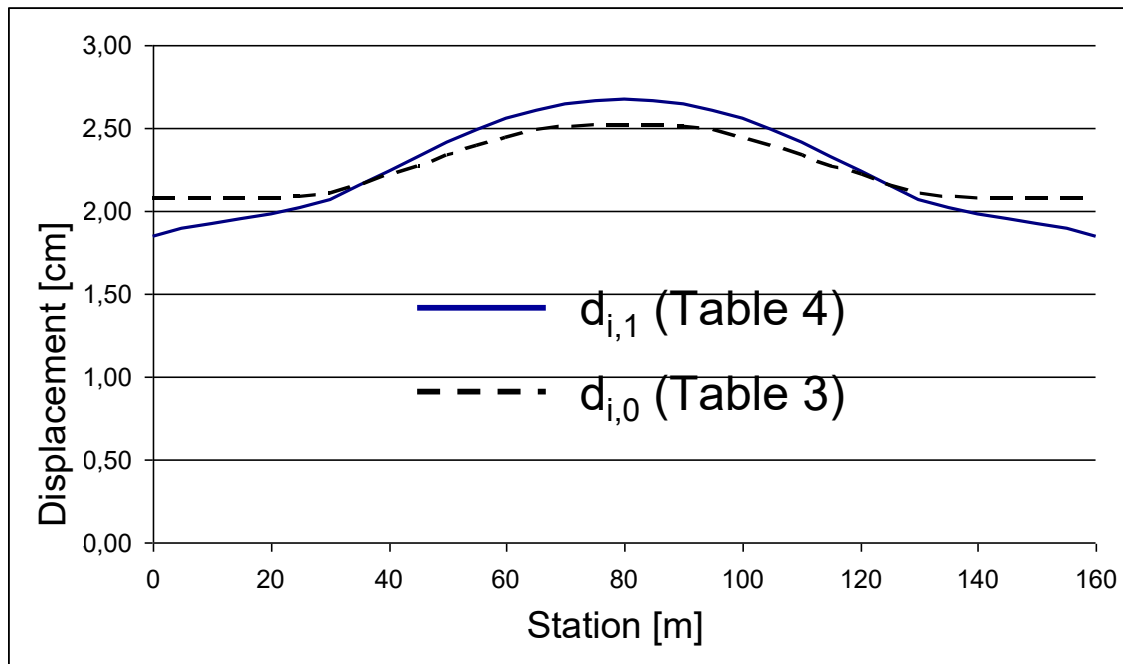
node	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
d_i [cm]	2,07	2,07	2,07	2,07	2,07	2,09	2,11	2,16	2,22	2,27	2,34	2,39	2,44	2,49	2,51	2,52	2,52
node	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	
d_i [cm]	2,52	2,51	2,49	2,44	2,39	2,34	2,27	2,22	2,16	2,11	2,09	2,07	2,07	2,07	2,07	2,07	

Displacements from Table 3 are compared with displacements $d_{i,1}$, corresponding to forces F_i (see Table 2)

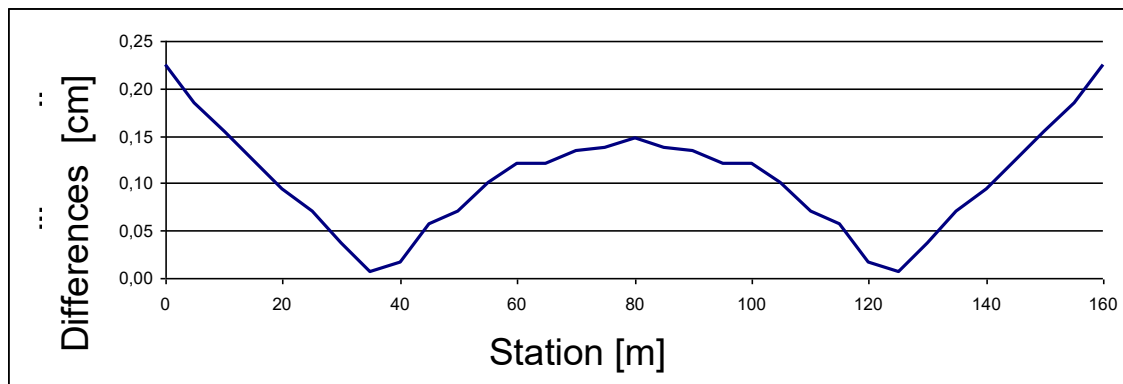
Table 4 displacements $d_{i,1}$

node	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
d_i [cm]	1,85	1,89	1,92	1,95	1,98	2,02	2,07	2,15	2,24	2,33	2,41	2,49	2,56	2,61	2,64	2,66	2,67
node	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	
d_i [cm]	2,66	2,64	2,61	2,56	2,49	2,41	2,33	2,24	2,15	2,07	2,02	1,98	1,95	1,92	1,89	1,85	

Displacements from Table 3 and 4

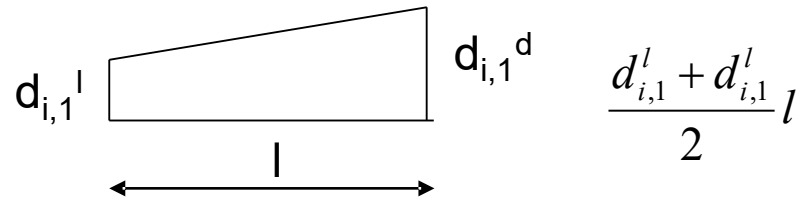


Differences between $d_{i,0}$ in $d_{i,1}$



1. Area P_d , corresponding to displacements $d_{i,1}$ is calculated
2. Area P_{Δ} , corresponding to differences between $d_{i,1}$ in $d_{i,0}$ is calculated
3. P_d and P_{Δ} are compared.

Areas P_d and P_{Δ} are calculated as it is illustrated in the following Figure



If $P_{\Delta}/P_d < 20\%$, the results of Fundamental mode method are acceptable. Otherwise the response spectrum analysis should be used.

For the analyzed bridge

$$P_d = 3,168 \text{ m}^2$$

$$P_{\Delta} = 0,161 \text{ m}^2$$

$P_{\Delta}/P_d = 4,4\% < 10\%$ regular structure – FMM can be used.

In EC8/2 it is required to take into account the torsional effects when FMM is used.

$M_t = F e$, where F is seismic force, e eccentricity

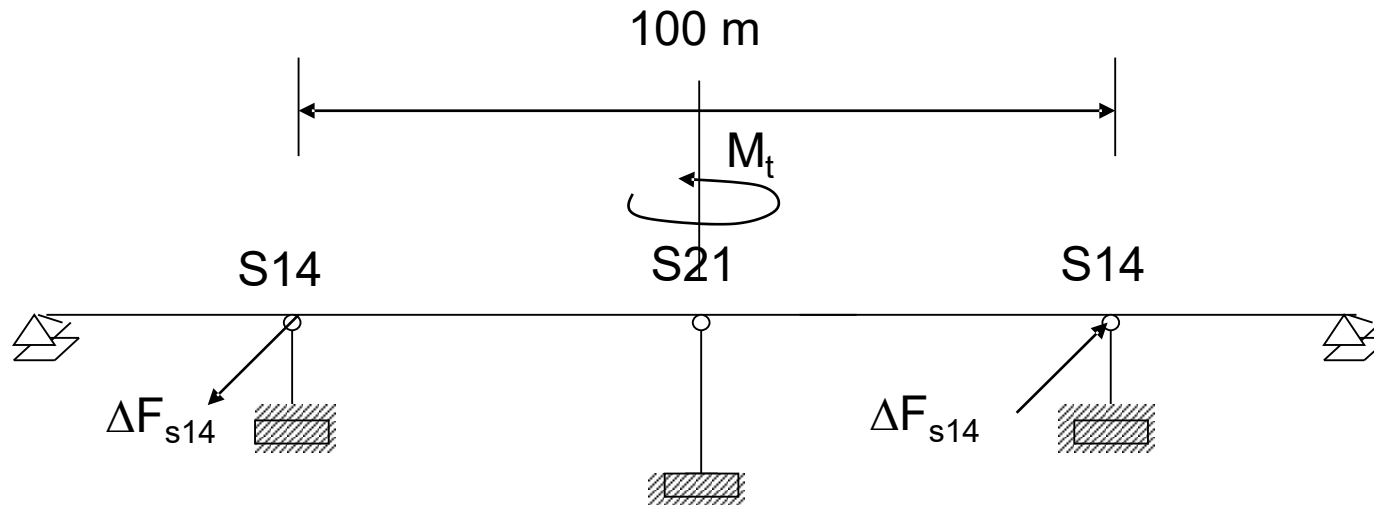
In the analyzed bridge $F = \sum F_i = 8149$ kN (sum of the forces from Table 2)

$e = e_0 + e_a = 0 + 0,05 L = 0,05 \cdot 160 = 8$ m

L is the length of the bridge

$M_t = 8149 \cdot 8 = 65192$ kNm

This moment is divided to columns supposing the rigid deck, as it is demonstrated in the following slide



$$\Delta F_{S14} = 65192 / 100 = 652 \text{ kN}$$

Final values of internal forces in S14 are

$$F_{S14} = 3408 + 652 = 4060 \text{ kN}$$

$$M_{S14} = F_{S14} \times H_s = 4060 \times 14 = 56840 \text{ kNm}$$

These forces are not considered in the further design, since the Response spectrum analysis (presented in the following slides) results in smaller demand, because the accidental eccentricity e_a should not be taken into account (only in very short and skewed bridges)

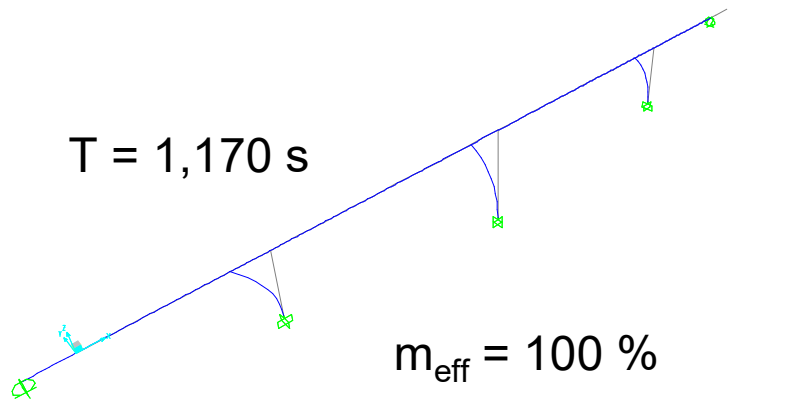
Response spectrum method (RSM)

In general the programme is needed to define the fundamental modes of vibration.

Modes of vibrations

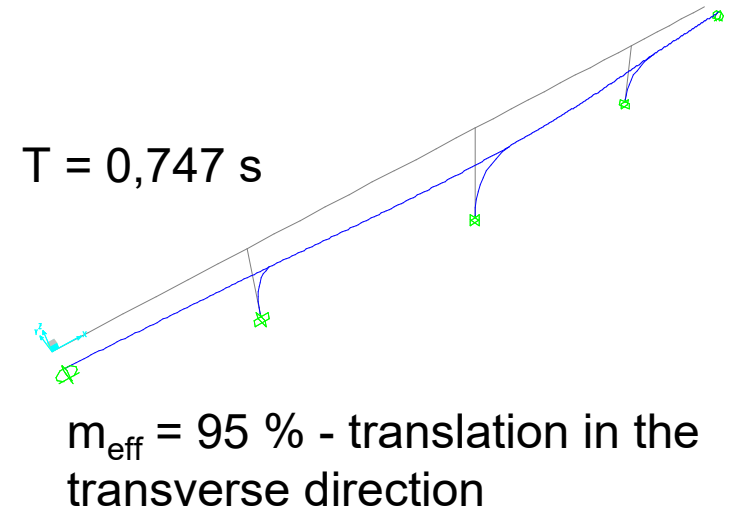
1.mode

1. Mode in the longitudinal direction



2. mode

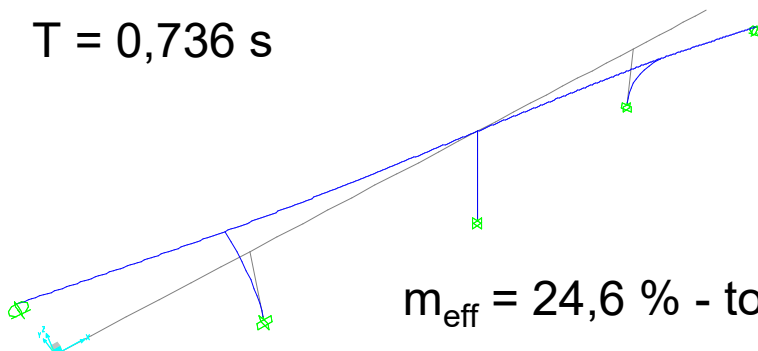
1. Mode in the transverse direction



$m_{\text{eff}} = 71,5\%$ - torsion

3. mode

Torsional mode – rotations around vertical axis



Note: Torsional mode is not activated, since the structure is symmetric and the masses are symmetric

In each direction Σm_{eff} should be at least 90% of the total mass

Comparison of the periods of vibrations defined by FMM and RSM

Longitudinal direction

FMM

$$T = 1,171 \text{ s}$$

RSM

$$T = 1,170 \text{ s}$$

Transverse direction

FMM

$$T = 0,742 \text{ s}$$

RSM

$$T = 0,747 \text{ s}$$

Spectral accelerations $S_d(T)$ corresponding to each mode of vibration are defined

Internal forces and displacements due to the each mode of vibration are defined

Contributions of different modes are combined using SRSS or CQC rule.

Shear forces in piers, defined using FMM and RSM

Longitudinal direction

FMM (without torsion)

$$V_{s14} = 2256 \text{ kN}$$

$$V_{s21} = 685 \text{ kN}$$

RSM

$$V_{s14} = 2261 \text{ kN}$$

$$V_{s21} = 688 \text{ kN}$$

Difference

$$0,22 \%$$

$$0,44 \%$$

Transverse direction

FMM (without torsion)

$$V_{s14} = 3408 \text{ kN}$$

$$V_{s21} = 1345 \text{ kN}$$

RSM

$$V_{s14} = 3256 \text{ kN}$$

$$V_{s21} = 1408 \text{ kN}$$

$$4,7\%$$

$$4,7\%$$

Displacements due to the seismic action

Effective stiffness (cross-section) of RC elements should be taken into account.

Effective moment of inertia I_{eff} can be defined according to Annex C

$$E_c I_{eff} = \frac{1,2 M_{Rd}}{\Phi_y}$$

$$\Phi_y = \frac{(\varepsilon_{sy} - \varepsilon_{cy})}{d_s}$$

M_{Rd} – design flexural strength

Φ_y – yield curvature

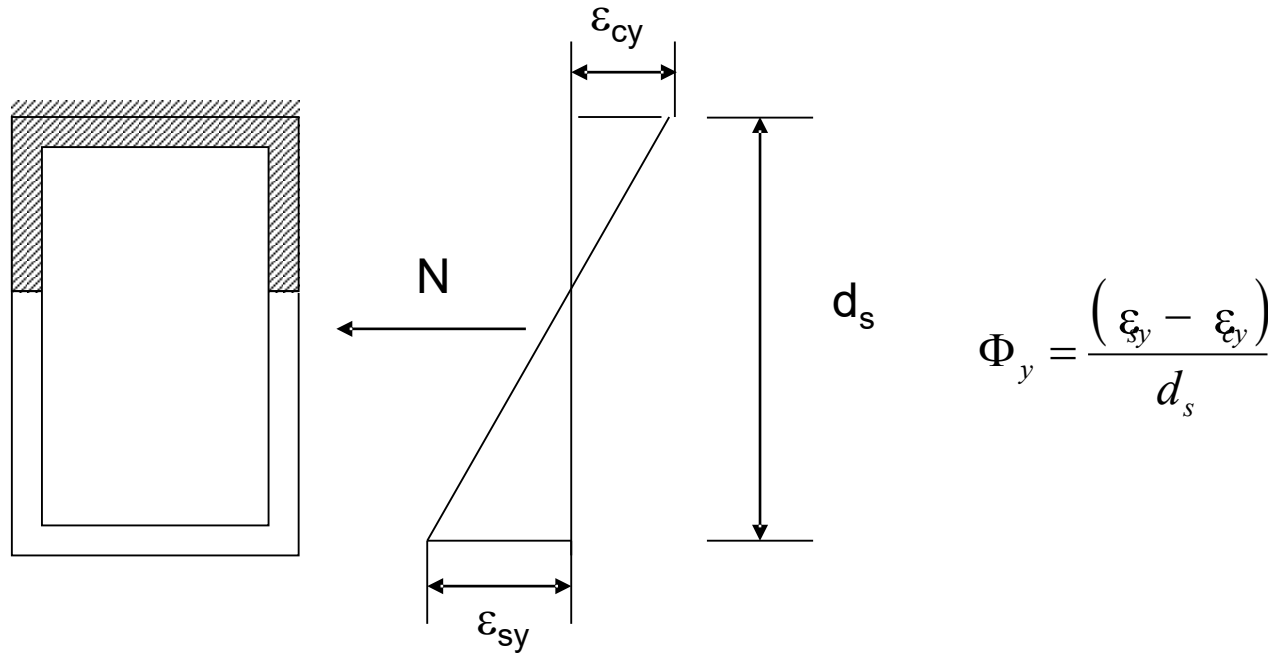
ε_{sy} – yield strain of the reinforcement

ε_{cy} – concrete compressive strain corresponding to yielding of the reinforcement

d_s - effective depth of the cross-section

Approximation of curvature Φ_y in rectangular cross-sections

$$\Phi_y = 2,1 \varepsilon_{sy} / d_s$$



$$\Phi_y = \frac{(\epsilon_{sy} - \epsilon_{cy})}{d_s}$$

To define the effective stiffness the design flexural strength M_{Rd} (flexural reinforcement) should be known.

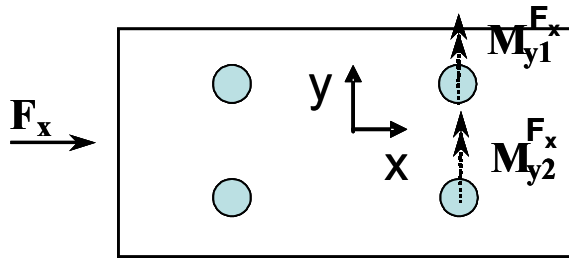
If the effective stiffness are used only to calculate the displacements, the flexural reinforcement in columns should be defined prior to the estimation of displacements.

If the seismic forces are also estimated based on the effective stiffness, M_{Rd} should be assumed. The assumption should be checked at the end of the analysis.

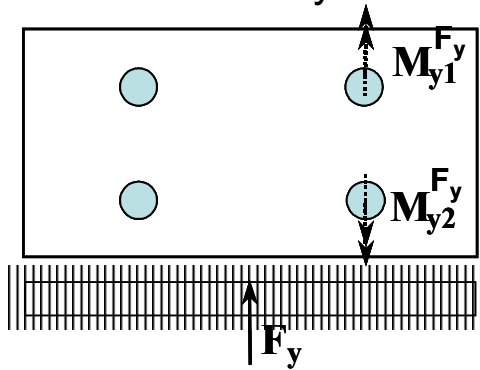
Bridge



Seismic action in the direction x



Seismic action in the direction y

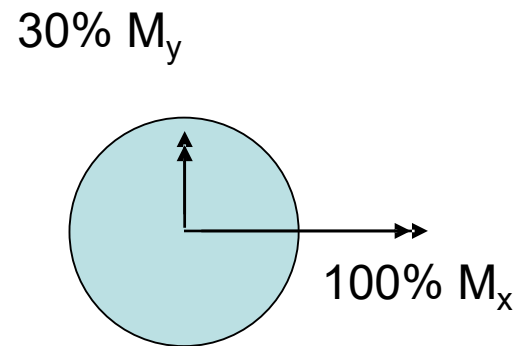
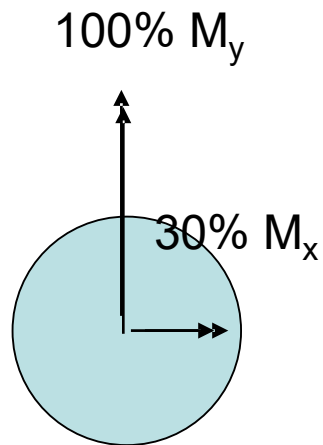


Plan view

$$M_{y1} = \sqrt{M_{y1,Fx}^2 + M_{y1,Fy}^2}$$

$$M_{y2} = \sqrt{M_{y2,Fx}^2 + M_{y2,Fy}^2}$$

Bi-axial bending should be taken into account when the flexural reinforcement of piers is defined



Estimation of the effective stiffness

The flexural reinforcement in piers is defined first

Results of RSM are considered

Pier S14

The basement of the pier

Transverse direction

$N = 14329 \text{ kN}$ (axial force due to the dead load)

$M_y = 45582 \text{ kNm}$

$M_z = 9496 \text{ kNm}$ (30% of the bending moment in the longitudinal direction)

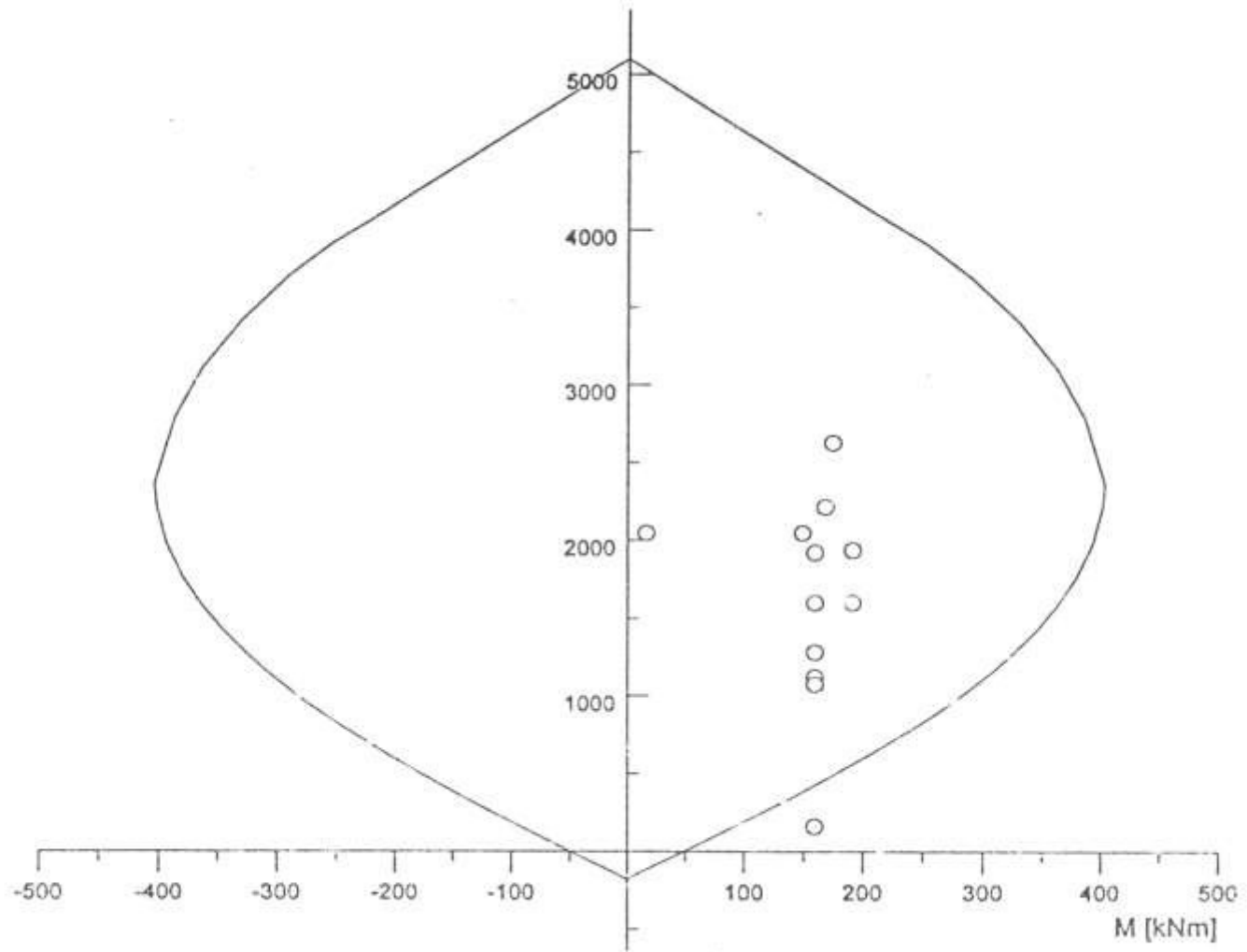
Longitudinal direction

$N = 14329 \text{ kN}$ (axial force due to the dead load)

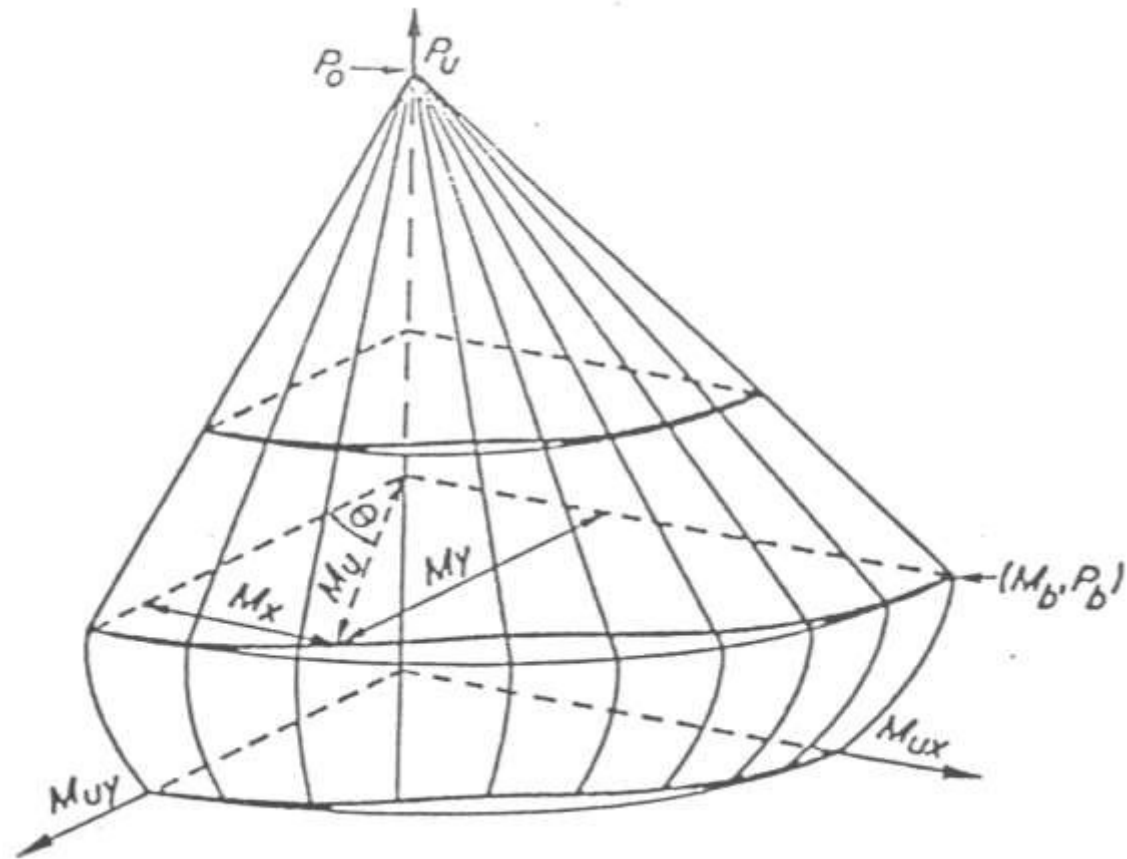
$M_y = 13675 \text{ kNm}$ (30% of the bending moment in the transverse direction)

$M_z = 31654 \text{ kNm}$

Interaction diagram – uniaxial bending



Interaction diagram – bi-axial bending (M_z - M_y - N)



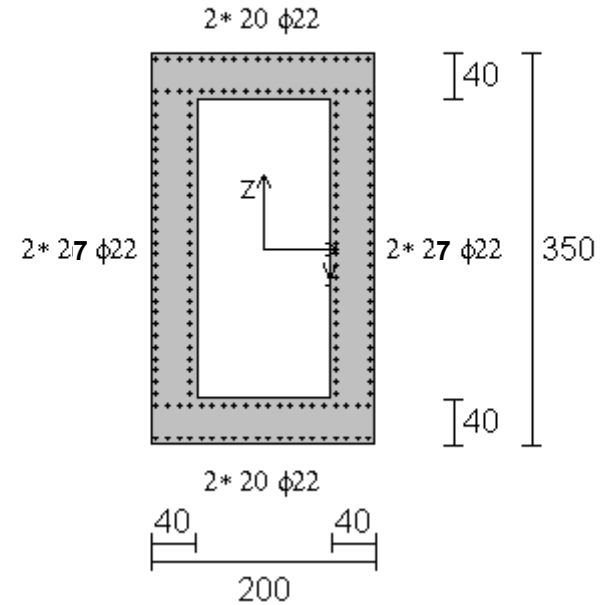
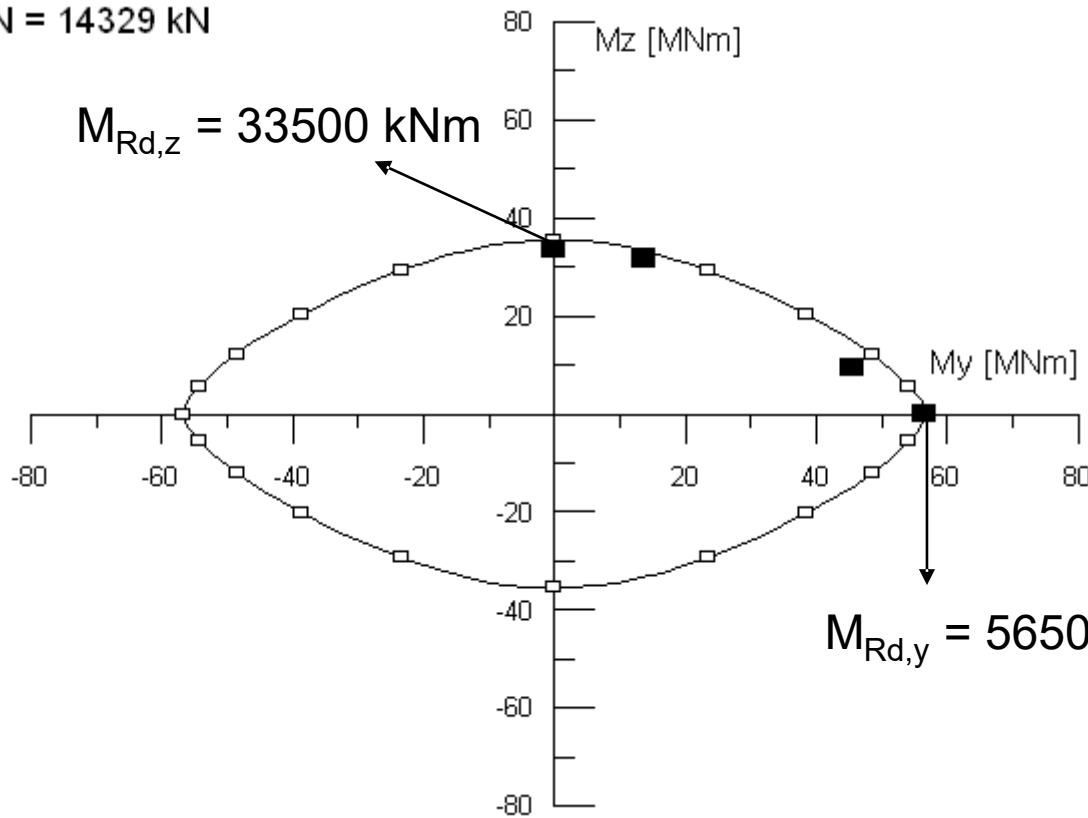
C 25/30
 σ_j 500 MPa

Arm. od roba 5.0 cm
 $\mu = 1.86\%$

$N = 14329$ kN

$M_{Rd,z} = 33500$ kNm

$M_{Rd,y} = 56500$ kNm



Effective moment of inertia

Results of RSM are taken into account

Pier S14

Transverse direction

$$\xi_y = \frac{f_{sy}}{E_s} = \frac{500 \cdot 1000 / 1,15}{2,0 \cdot 10^8} = 0,00217$$

$$\Phi_y = 2,1 \xi_y / d_s = 2,1 \frac{0,00217}{0,9 \cdot 3,5} = 0,00145$$

$$E_c I_{eff} = \frac{1,2 M_{Rd}}{\Phi_y} = \frac{1,2 \cdot 56500}{0,00145} = 46758600 kNm^2$$

$$I_{eff,y} = 1,51 m^4$$

$$\frac{I_{eff,y}}{I_y} = \frac{1,51}{5,18} = 0,29$$

Longitudinal direction

$$\xi_y = \frac{f_{sy}}{E_s} = \frac{500 \cdot 1000 / 1,15}{2,0 \cdot 10^8} = 0,00217$$

$$\Phi_y = 2,1 \xi_y / d_s = 2,1 \frac{0,00217}{0,9 \cdot 2} = 0,00253$$

$$E_c I_{eff} = \frac{1,2 M_{Rd}}{\Phi_y} = \frac{1,2 \cdot 33500}{0,00253} = 15889330 kNm^2$$

$$I_{eff,z} = 0,51 m^4$$

$$\frac{I_{eff,z}}{I_z} = \frac{0,51}{1,94} = 0,26$$

Shear areas are also appropriately reduced

Pier S21

The basement of the pier

Transverse direction

$N = 17485 \text{ kN}$ (axial force due to the dead load)

$M_y = 29567 \text{ kNm}$

$M_z = 4332 \text{ kNm}$ (30% of the bending moment in the longitudinal direction)

Longitudinal direction

$N = 17485 \text{ kN}$ (axial force due to the dead load)

$M_y = 8870 \text{ kNm}$ (30% of the bending moment in the transverse direction)

$M_z = 14440 \text{ kNm}$

Note:

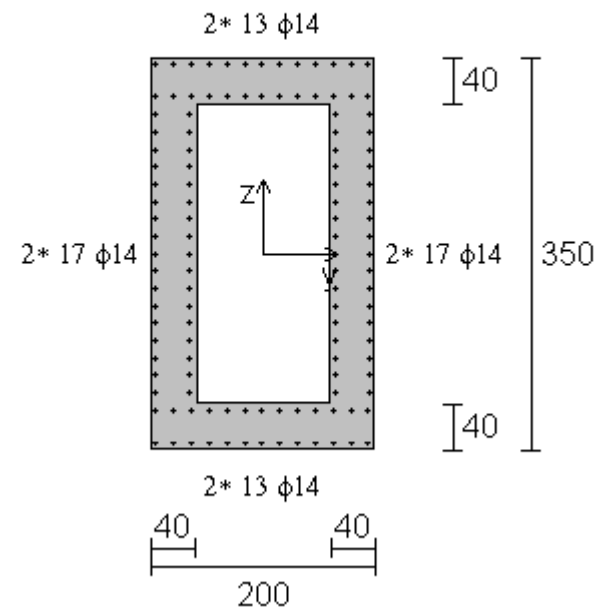
In EC8/1 minimum flexural reinforcement in columns amounts to 1%. The response of hollow box cross-sections is similar to that of the walls. Therefore, the minimum flexural reinforcement of 0,5% was taken into account. This is the minimum reinforcement required in flanges of the walls with limited ductile response - EC8/1

C 25/30

σ_j 500 MPa

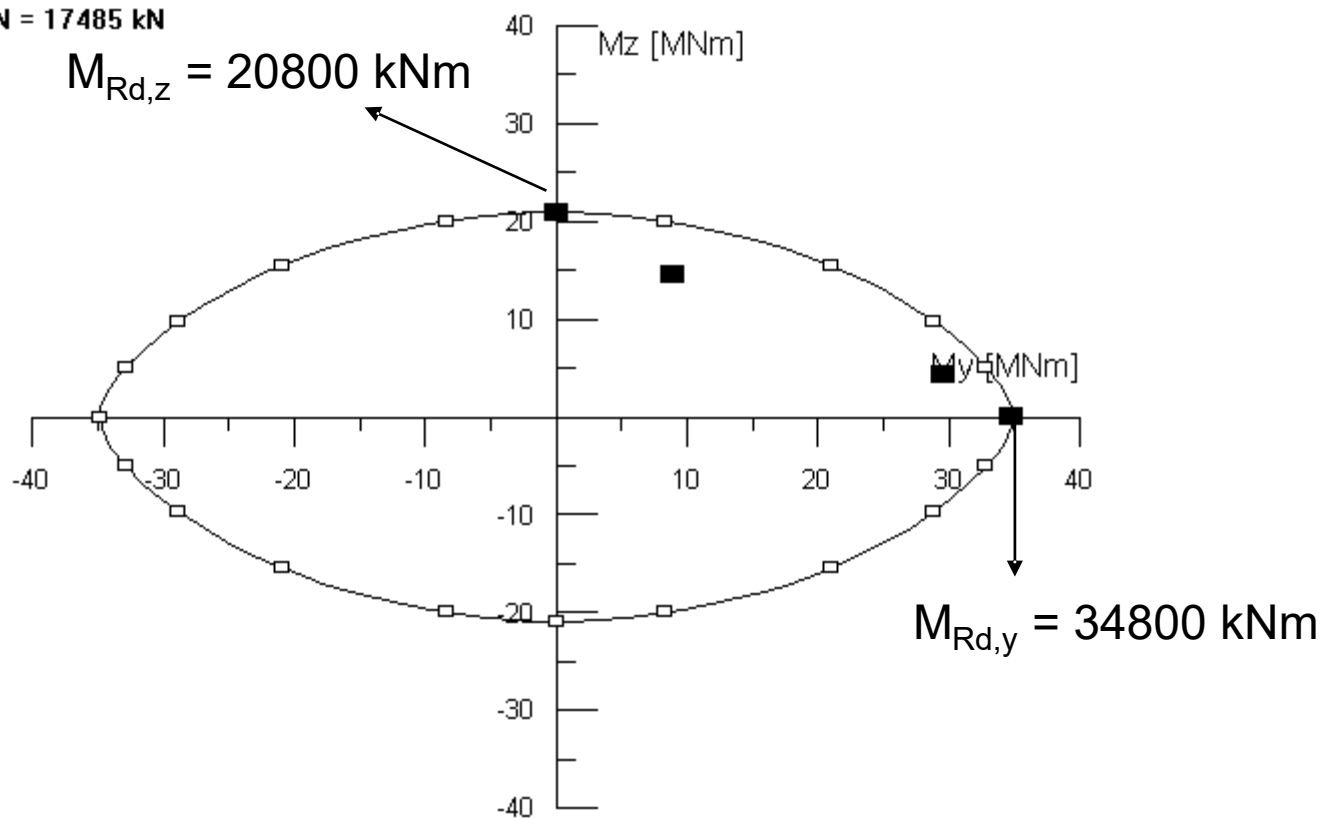
Arm. od roba 5.0 cm

$\mu = 0.49\%$



$N = 17485$ kN

$M_{Rd,z} = 20800$ kNm



Effective moment of inertia

Results of RSM are taken into account

Pier S21

Transverse direction

$$\varepsilon_{sy} = \frac{f_{sy}}{E_s} = \frac{500 \cdot 1000 / 1,15}{2,0 \cdot 10^8} = 0,00217$$

$$\Phi_y = 2,1 \varepsilon_{sy} / d_s = 2,1 \frac{0,00217}{0,9 \cdot 3,5} = 0,00145$$

$$E_c I_{eff} = \frac{1,2 M_{Rd}}{\Phi_y} = \frac{1,2 \cdot 34800}{0,00145} = 288000000 kNm^2$$

$$I_{eff,y} = 0,93 m^4$$

$$\frac{I_{eff,y}}{I_y} = \frac{0,93}{5,18} = 0,18$$

Longitudinal direction

$$\varepsilon_{sy} = \frac{f_{sy}}{E_s} = \frac{500 \cdot 1000 / 1,15}{2,0 \cdot 10^8} = 0,00217$$

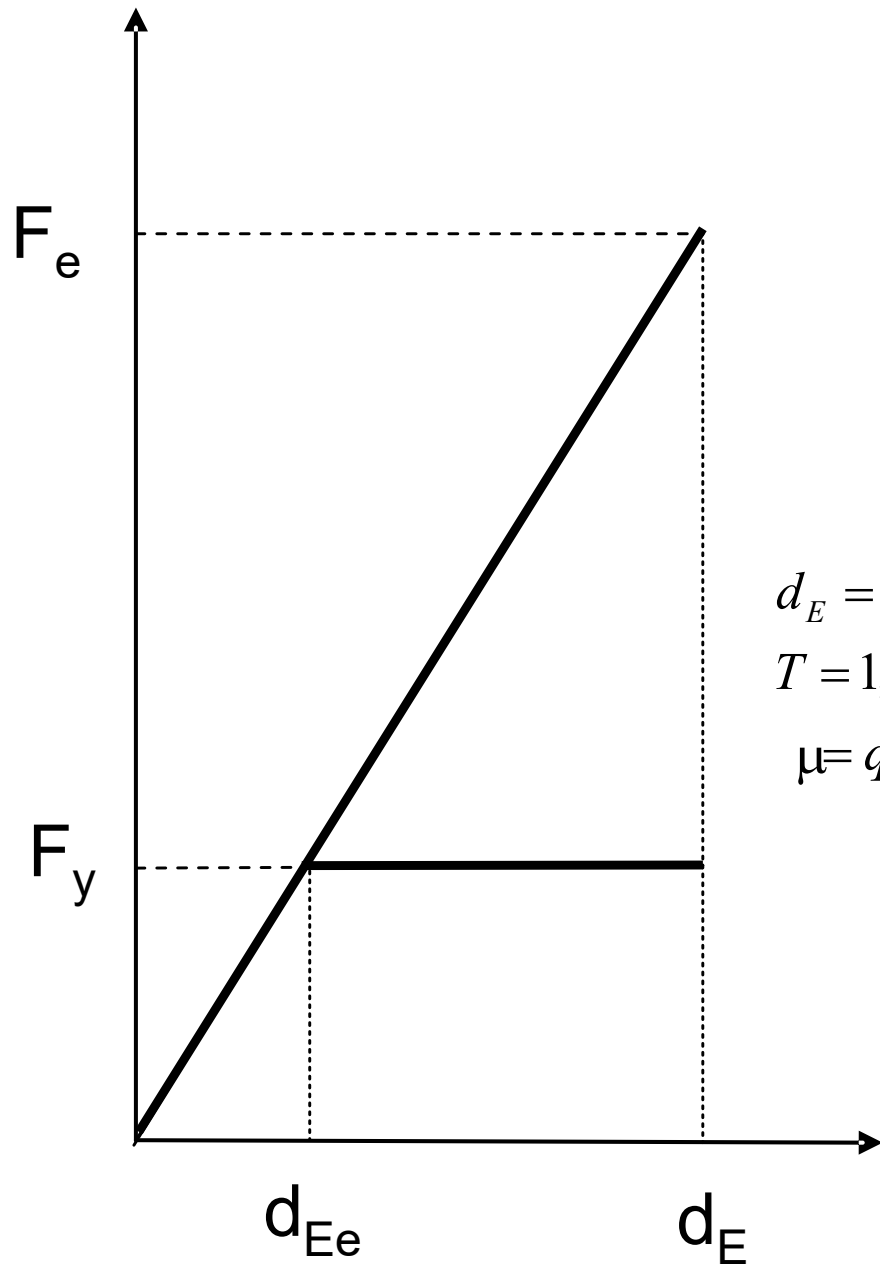
$$\Phi_y = 2,1 \varepsilon_{sy} / d_s = 2,1 \frac{0,00217}{0,9 \cdot 2} = 0,00253$$

$$E_c I_{eff} = \frac{1,2 M_{Rd}}{\Phi_y} = \frac{1,2 \cdot 20800}{0,00253} = 9865610 kNm^2$$

$$I_{eff,z} = 0,32 m^4$$

$$\frac{I_{eff,z}}{I_z} = \frac{0,32}{1,94} = 0,16$$

Shear areas are also appropriately reduced



$$d_E = \mu d_{Ee}$$

$$T = 1,376s > 1,2T_C = 1,2 \cdot 0,6 = 0,72s$$

$$\mu = q = 3,5$$

Analysis is repeated taking into account estimated effective stiffness

Periods of vibrations corresponding to the effective stiffness

$$T_1 = 2,355 \text{ s} \quad m_{\text{eff}} = 100\%$$

$$T_2 = 1,376 \text{ s} \quad m_{\text{eff}} = 99,6\% \quad (m_{\text{eff}} = 75\% - \text{zasuki okoli z osi})$$

$$T_3 = 1,288 \text{ s} \quad m_{\text{eff}} = 25\% \quad (\text{zasuki okoli z osi})$$

Displacements above the abutments:

Displacements corresponding to the reduced seismic action

Longitudinal direction

$$d_{\text{Ee,long}} = 6,12 \text{ cm} \quad (T_1 > 2\text{s} \text{ displacements are defined based on the elastic spectrum})$$

Transverse direction

$$d_{\text{Ee,tran}} = 3,80 \text{ cm}$$

Displacements due to the seismic action:

Longitudinal direction

$$d_E = \mu d_{\text{Ee}} = q d_{\text{Ee}} = 3,5 \cdot 6,12 = 21,4 \text{ cm}$$

Transverse direction

$$d_E = \mu d_{\text{Ee}} = q d_{\text{Ee}} = 3,5 \cdot 3,8 = 13,3 \text{ cm}$$

Ragularity of the bridge

$$r = q M_{Ed} / M_{Rd}$$

In piers S14 $r = q = 3,5$, since the flexural strength is fully exploited

Pier S21 can be neglected, since its contribution to total base shear is less than 20%

Longitudinal direction

$$V_{ES,21} = 688 \quad V_{tot} = 5210 \quad V_{ES,21}/V_{tot} = 688 / 5210 = 0,132$$

Transverse direction

$$V_{ES,21} = 1408 \quad V_{tot} = 7920 \quad V_{ES,21}/V_{tot} = 1408 / 7920 = 0,178$$

$$r_{max} = r_{min} \Rightarrow \rho = 1 < \rho_o = 2 \quad \text{bridge is regular}$$

The presented procedure, used to estimate the seismic action effects in terms of forces is conservative

They can be also estimated considering the effective stiffness

To estimate the effective stiffness the flexural strength M_{Rd} (flexural reinforcement) should be assumed

According to EC8/1 the effective stiffness can be estimated reducing the stiffness corresponding to the gross cross-sections by 50%. Thus the moments of inertia and shear areas corresponding to gross cross-sections were reduced by 50%.

The corresponding periods of vibrations were

$$T_1 = 1,654 \text{ s} \quad m_{\text{eff}} = 100\%$$

$$T_2 = 1,038 \text{ s} \quad m_{\text{eff}} = 99,5\% \quad (m_{\text{eff}} = 74,9\% - \text{torsion})$$

$$T_3 = 1,014 \text{ s} \quad m_{\text{eff}} = 24,7\% \text{ (torsion)}$$

Internal forces in piers

Pier S14

Internal forces at the basement

Transverse direction

$N = 14329$ kN (axial force due to the dead load)

$M_y = 34771$ kNm

$M_z = 6713$ kNm (30% of the bending moment in the longitudinal direction)

Longitudinal direction

$N = 14329$ kN (axial force due to the dead load)

$M_y = 10431$ kNm (30% of the bending moment in the transverse direction)

$M_z = 22377$ kNm

Internal forces in piers

Pier S21

Internal forces at the basement

Transverse direction

$N = 17485 \text{ kN}$ (axial force due to the dead load)

$M_y = 18494 \text{ kNm}$

$M_z = 3059 \text{ kNm}$ (30% of the bending moment in the longitudinal direction)

Longitudinal direction

$N = 17485 \text{ kN}$ (axial force due to the dead load)

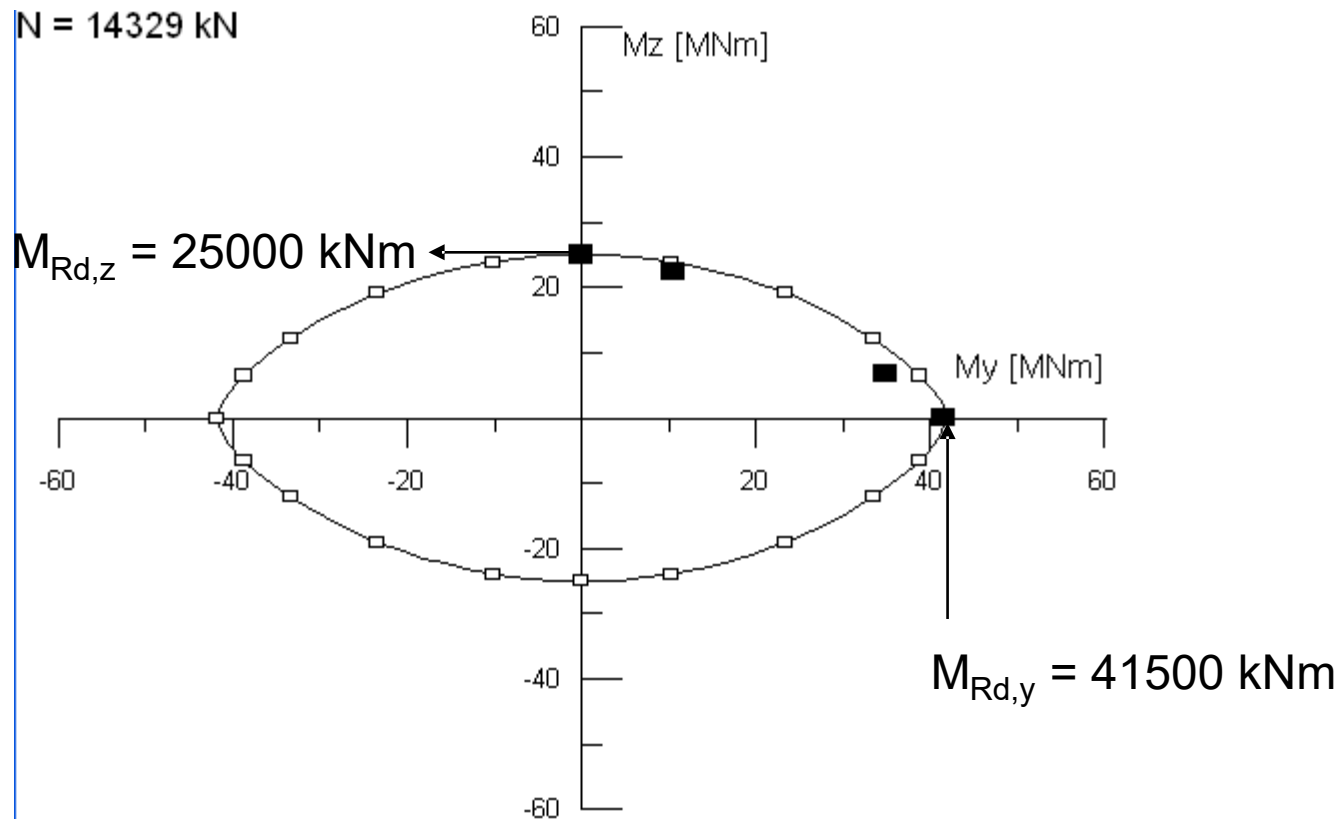
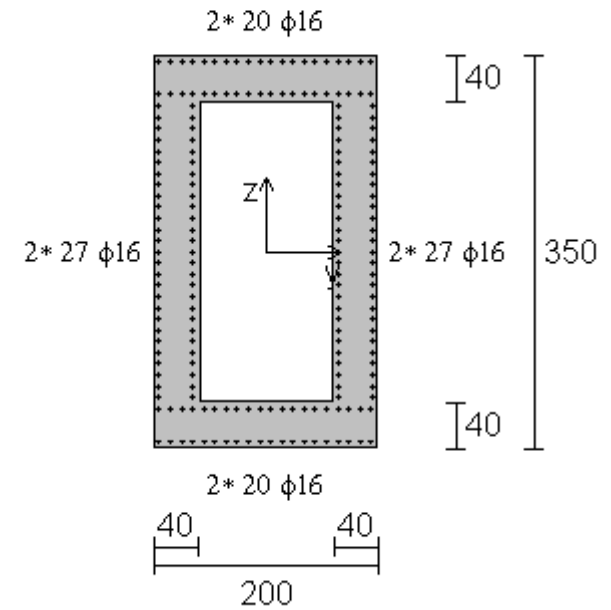
$M_y = 5548 \text{ kNm}$ (30% of the bending moment in the transverse direction)

$M_z = 10197 \text{ kNm}$

C 25/30
 σ_j 500 MPa

Arm. od roba 5.0 cm
 $\mu = 1.01\%$

Flexural reinforcement in pier S14



Effective moments of inertia

Pier S14

Transverse direction

$$\xi_{sy} = \frac{f_{sy}}{E_s} = \frac{500 \cdot 1000 / 1,15}{2,0 \cdot 10^8} = 0,00217$$

$$\Phi_y = 2,1 \xi_{sy} / d_s = 2,1 \frac{0,00217}{0,9 \cdot 3,5} = 0,00145$$

$$E_c I_{eff} = \frac{1,2 M_{Rd}}{\Phi_y} = \frac{1,2 \cdot 41500}{0,00145} = 34344827 kNm^2$$

$$I_{eff,y} = 1,11 m^4$$

$$\frac{I_{eff,y}}{I_y} = \frac{1,11}{5,18} = 0,21$$

Longitudinal direction

$$\xi_{sy} = \frac{f_{sy}}{E_s} = \frac{500 \cdot 1000 / 1,15}{2,0 \cdot 10^8} = 0,00217$$

$$\Phi_y = 2,1 \xi_{sy} / d_s = 2,1 \frac{0,00217}{0,9 \cdot 2} = 0,00253$$

$$E_c I_{eff} = \frac{1,2 M_{Rd}}{\Phi_y} = \frac{1,2 \cdot 25000}{0,00253} = 11857710 kNm^2$$

$$I_{eff,z} = 0,38 m^4$$

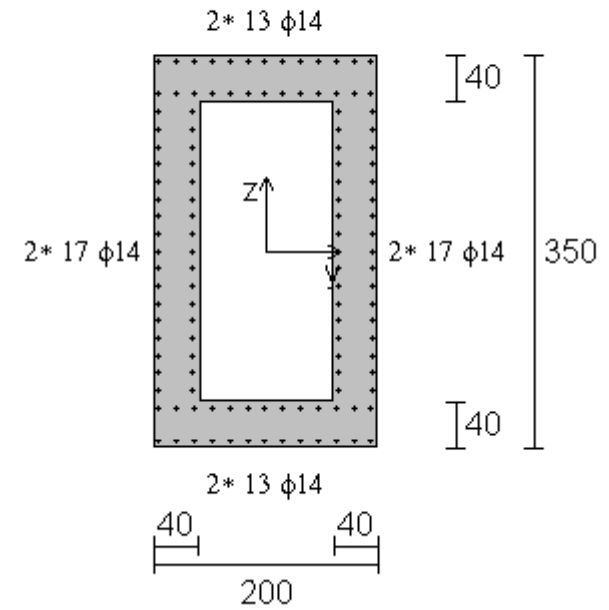
$$\frac{I_{eff,z}}{I_z} = \frac{0,38}{1,94} = 0,20$$

Shear areas are also appropriately reduced

C 25/30
 σ_j 500 MPa

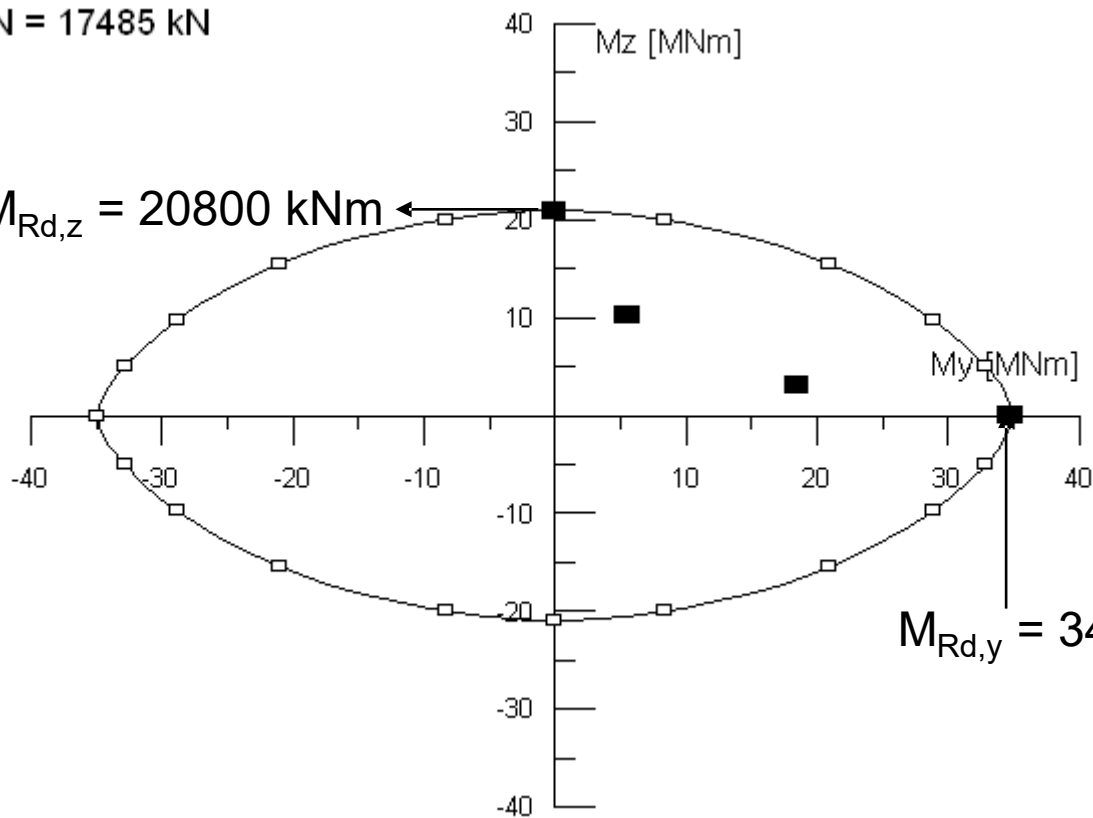
Arm. od roba 5.0 cm
 $\mu = 0.49\%$

Flexural reinforcement in pier S21



$N = 17485$ kN

$M_{Rd,z} = 20800$ kNm



$M_{Rd,y} = 34800$ kNm

Effective moments of inertia

Pier S21

Transverse direction

$$\xi_{sy} = \frac{f_{sy}}{E_s} = \frac{500 \cdot 1000 / 1,15}{2,0 \cdot 10^8} = 0,00217$$

$$\Phi_y = 2,1 \xi_{sy} / d_s = 2,1 \frac{0,00217}{0,9 \cdot 3,5} = 0,00145$$

$$E_c I_{eff} = \frac{1,2 M_{Rd}}{\Phi_y} = \frac{1,2 \cdot 34800}{0,00145} = 288000000 kNm^2$$

$$I_{eff,y} = 0,93 m^4$$

$$\frac{I_{eff,y}}{I_y} = \frac{0,93}{5,18} = 0,18$$

Longitudinal direction

$$\xi_{sy} = \frac{f_{sy}}{E_s} = \frac{500 \cdot 1000 / 1,15}{2,0 \cdot 10^8} = 0,00217$$

$$\Phi_y = 2,1 \xi_{sy} / d_s = 2,1 \frac{0,00217}{0,9 \cdot 2} = 0,00253$$

$$E_c I_{eff} = \frac{1,2 M_{Rd}}{\Phi_y} = \frac{1,2 \cdot 20800}{0,00253} = 9865610 kNm^2$$

$$I_{eff,z} = 0,32 m^4$$

$$\frac{I_{eff,z}}{I_z} = \frac{0,32}{1,94} = 0,16$$

Shear areas are also appropriately reduced

The effective stiffness of pier S14 is smaller than that assumed at the beginning of the analysis. Estimated forces are still conservative. Displacements should be estimated based on the reduced effective stiffness.

The effective stiffness of Pier S21 is also smaller than the assumed value.

However it is not clear that the estimated effective stiffness will be actually activated since the provided strength is larger from the required value (minimum reinforcement is provided). Therefore, the reduction of the effective stiffness larger than 50% can not be taken into account for the estimation of forces. For the estimation of displacements the larger reduction of the effective stiffness can be taken into account since the results are conservative.

Displacements (estimated reduction of the effective stiffness was considered)

Periods of vibrations

$$T_1 = 2,642 \text{ s}$$

$$T_2 = 1,646 \text{ s}$$

$$T_3 = 1,557 \text{ s}$$

Displacements above the abutments d_{Ee} due to the reduced seismic action

Longitudinal direction

$$d_{Ee} = 6,12 \text{ cm (elastic spectrum was considered)}$$

Transverse direction

$$d_{Ee} = 4,82 \text{ cm}$$

Displacements due to the seismic action

Longitudinal direction

$$d_E = d_{Ee} \cdot q = 6,4 \cdot 3,5 = 21,4 \text{ cm}$$

Transverse direction

$$d_E = d_{Ee} \cdot q = 4,82 \cdot 3,5 = 16,9 \text{ cm}$$

Comparison of the two approaches

Periods of vibrations

Unreduced effective stiffness (forces)

$$T1 = 1,170 \text{ s}$$

$$T2 = 0,747 \text{ s}$$

Reduced effective stiffness (50%)

$$T1 = 1,654 \text{ s}$$

$$T2 = 1,038 \text{ s}$$

Flexural reinforcement

Unreduced effective stiffness (forces)
= 1,86%

Reduced effective stiffness (50%) S14 μ
S14 μ = 1,01%

S21 μ = 0,50% (minimum)

S21 μ = 0,50% (minimum)

Displacements (reduced eff. stiffness)

Displacements (reduced eff. stiffness)

$$d_{e,long} = 21,4 \text{ cm}$$

$$d_{e,long} = 21,4 \text{ cm}$$

$$d_{e,tran} = 13,3 \text{ cm}$$

$$d_{e,tran} = 16,9 \text{ cm}$$

Displacements due to the other actions

Displacements due to the seismic action should be combined with displacements due to the other actions:

- a) displacements d_G due to the permanent and quasi-permanent actions (e.g. pre-stressing, creep, shrinkage)
- b) Due to the quasi-permanent temperature effects $\psi_2 d_T$

Detailing

Flexural strength at the basement

Overstrength factor γ_o

Pier S14

$$\eta = \frac{N_{Ed}}{A_c f_{ck}} = \frac{14329}{3,76 \cdot 25 \cdot 10^3} = 0,152 < 0,2$$

$$\gamma_o = 1,35$$

Pier S21

$$\eta = \frac{N_{Ed}}{A_c f_{ck}} = \frac{17485}{3,76 \cdot 25 \cdot 10^3} = 0,186 < 0,2$$

$$\gamma_o = 1,35$$

Opomba:

Normalized axial force is less than 0,2; Special confinement reinforcement is not required
6.2.4(4)

Therefore, the overstrength factor amounts to 1,35

Flexural strength at the basement

Pier S14

Transverse direction

$$M_{Rd} = 56500 \text{ kNm}$$

$$M_o = \gamma_0 M_{Rd} = 1,35 \cdot 56500 = 76275 \text{ kNm}$$

Longitudinal direction

$$M_{Rd} = 33500 \text{ kNm}$$

$$M_o = \gamma_0 M_{Rd} = 1,35 \cdot 33500 = 45225 \text{ kNm}$$

Pier S21

Transverse direction

$$M_{Rd} = 34800 \text{ kNm}$$

$$M_o = \gamma_0 M_{Rd} = 1,35 \cdot 34800 = 46980 \text{ kNm}$$

Longitudinal direction

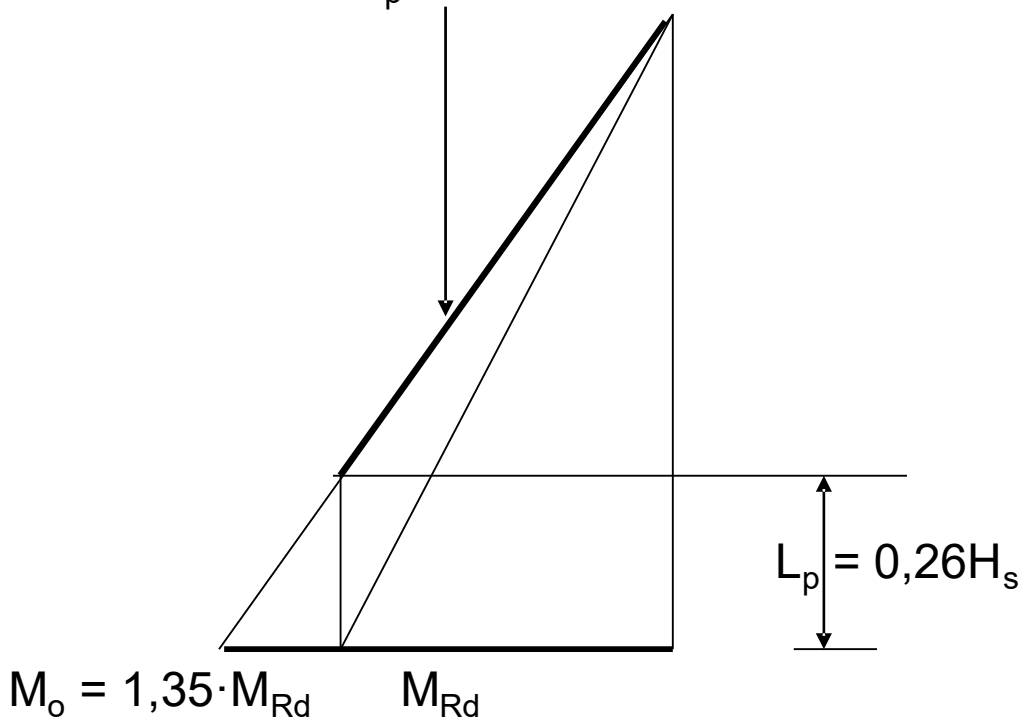
$$M_{Rd} = 20800 \text{ kNm}$$

$$M_o = \gamma_0 M_{Rd} = 1,35 \cdot 20800 = 28080 \text{ kNm}$$

Plastic hinge length and the flexural reinforcement outside the plastic hinge region

Bending moments that were taken into account to define the flexural reinforcement outside the plastic hinge region

Capacity design bending moments outside the L_p



H_s – height of the column

Minimum plastic hinge length:

$0,2 H_s$ or depth of the cross-section in the relevant direction (larger value)

In the analyzed bridge the minimum length of the plastic hinge is $L_p = 3,5\text{m}$

In pier S14 L_p is:

$$L_p = 0,26 \cdot 1400 = 364 \text{ cm}$$

In pier S21 L_p is:

$$L_p = 0,26 \cdot 2100 = 546 \text{ cm}$$

Maximum bending moments outside the plastic hinge region:

Pier S14

Transverse direction

$$M_y = 56500 \text{ kNm} \quad (N_G = 13987 \text{ kN})$$

Longitudinal direction

$$M_z = 33500 \text{ kNm} \quad (N_G = 13987 \text{ kN})$$

Pier S21

Transverse direction

$$M_y = 34800 \text{ kNm} \quad (N_G = 16972 \text{ kN})$$

Longitudinal direction

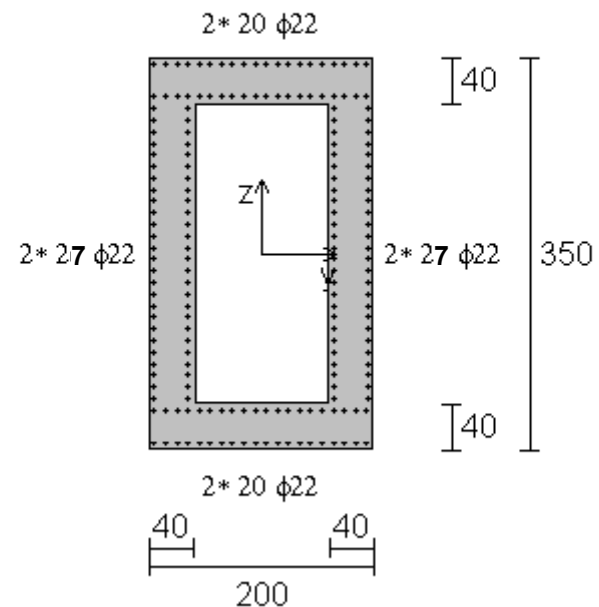
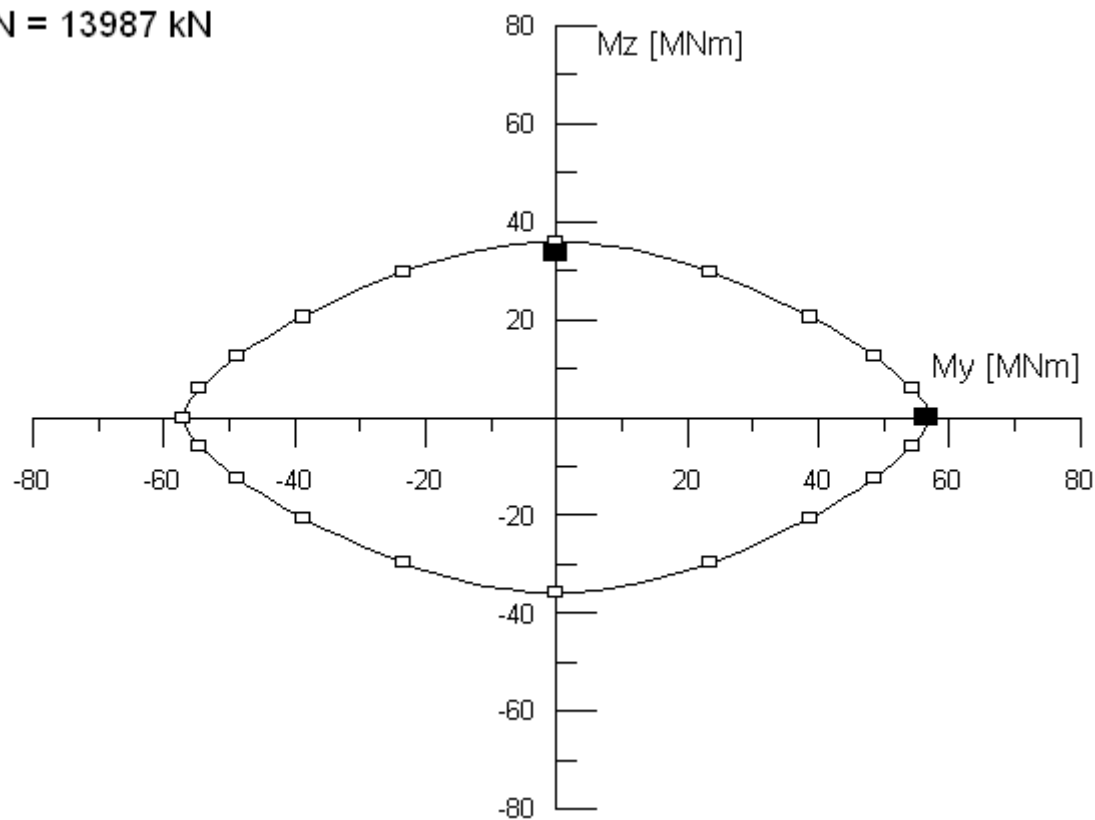
$$M_z = 20800 \text{ kNm} \quad (N_G = 16972 \text{ kN})$$

Pier S14

C 25/30
 σ_j 500 MPa

Arm. od roba 5.0 cm
 $\mu = 1.86\%$

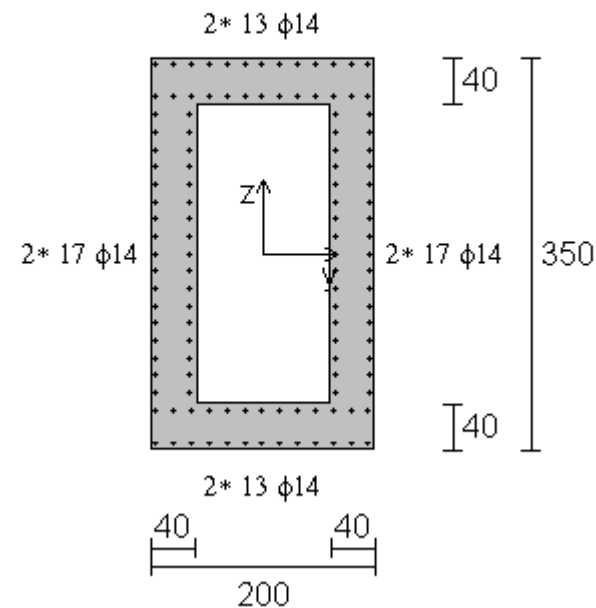
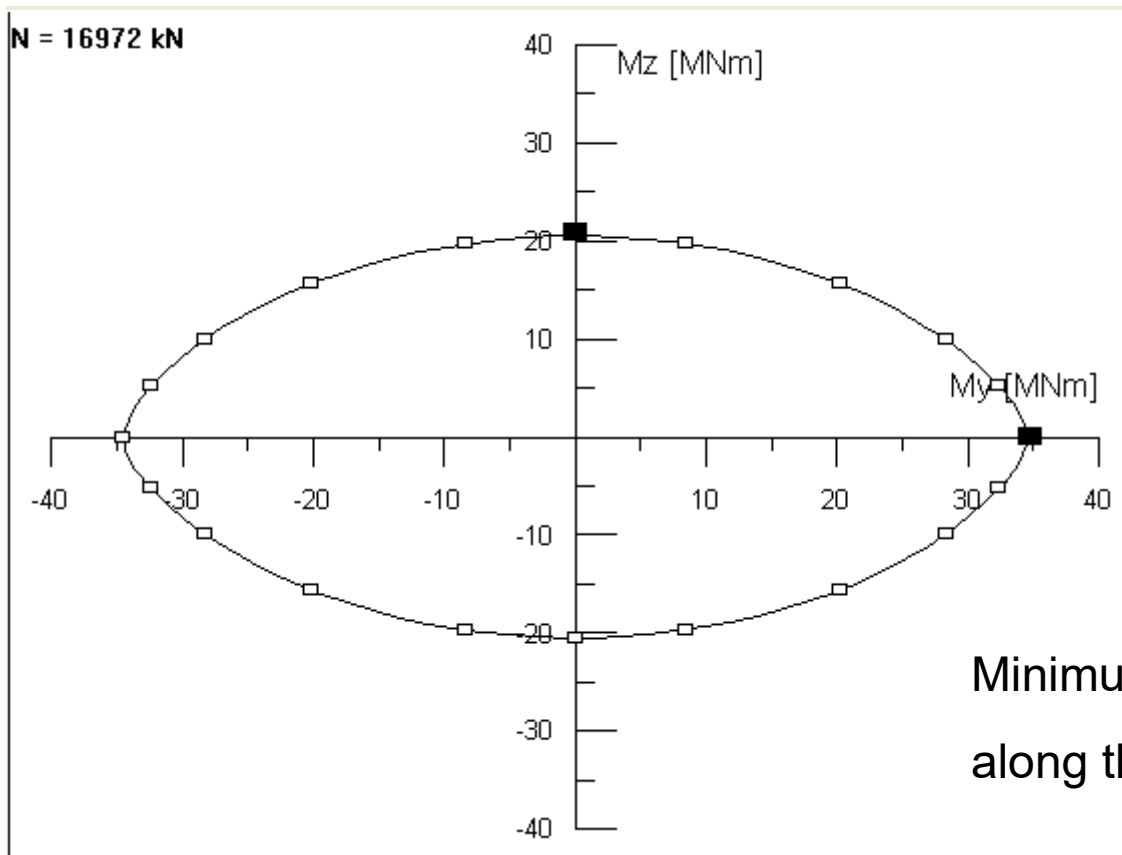
N = 13987 kN



Pier S21

C 25/30
 σ_j 500 MPa

Arm. od roba 5.0 cm
 $\mu = 0.49\%$



Minimum reinforcement
along the total height of pier S21

At the top 5,7 m of S14 minimum reinforcement provides adequate flexural strength

$$N_G = 13548 \text{ kN}$$

$$M_y = 5,7 / 14 \times 76275 = 31050 \text{ kNm}$$

$$M_z = 5,7 / 14 \times 45225 = 18410 \text{ kNm}$$

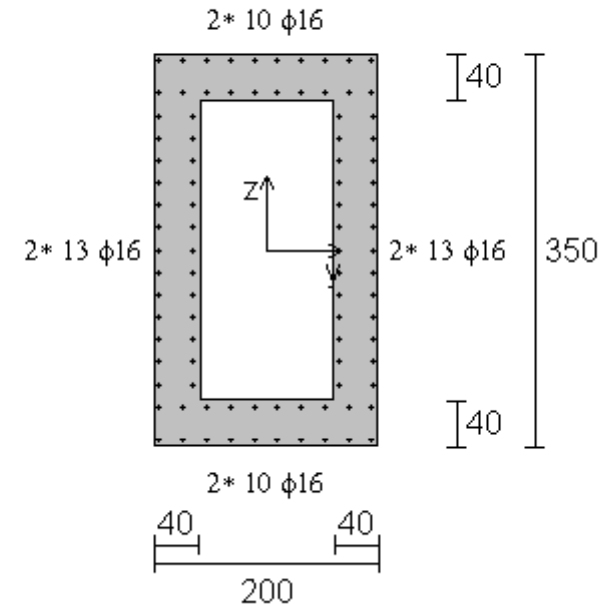
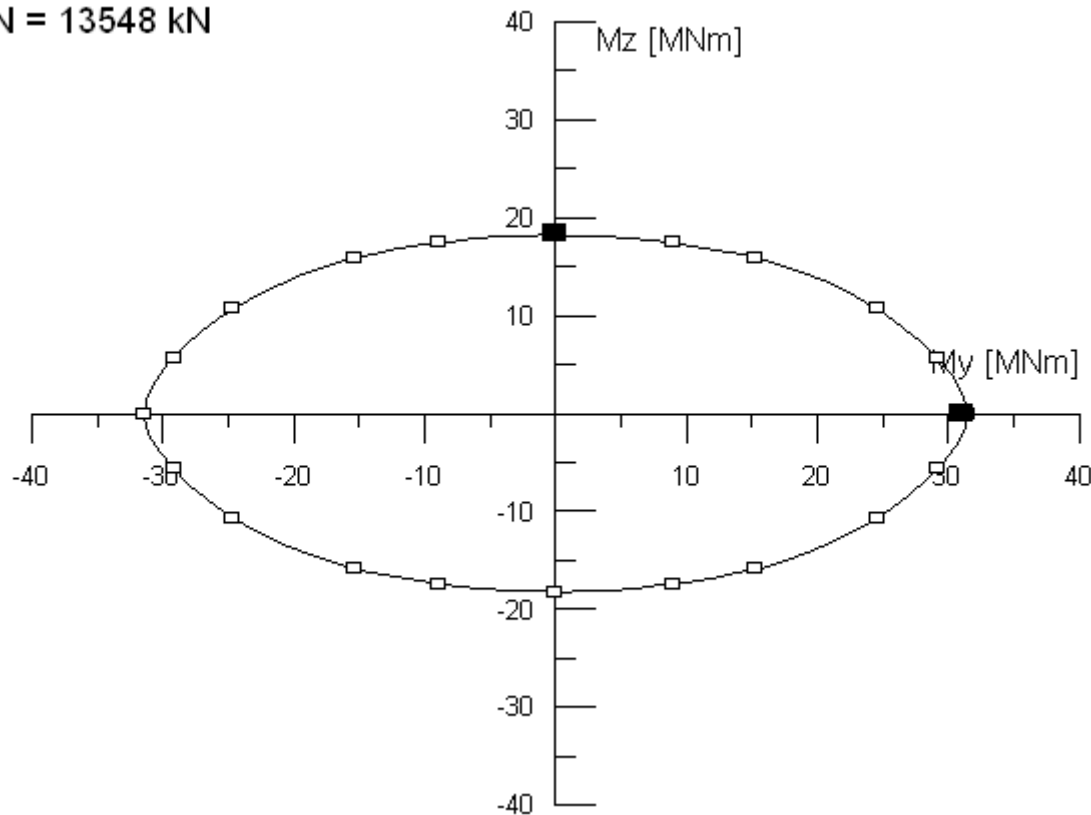
C 25/30

σ_j 500 MPa

Arm. od roba 5.0 cm

$\mu = 0.49\%$

$N = 13548 \text{ kN}$



Shear reinforcement at the plastic hinge region

Calculated based on the maximum shear forces V_c

In cantilever columns it is defined as: $V_c = \frac{M_o}{H_s}$

Pier S14

Transverse

$$V_c = \frac{M_o}{H_s} = \frac{76275}{14} = 5448kN$$

Longitudinal

$$V_c = \frac{M_o}{H_s} = \frac{45225}{14} = 3230kN$$

Pier S21

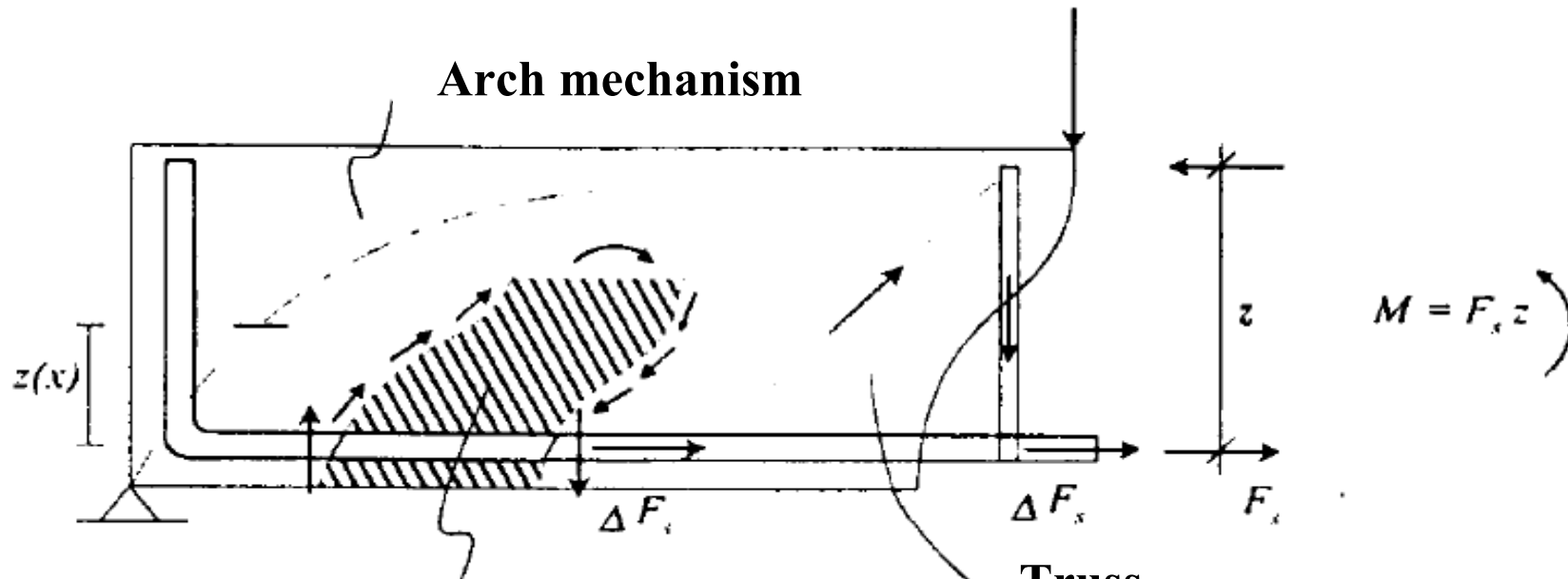
Transverse

$$V_c = \frac{M_o}{H_s} = \frac{46980}{21} = 2237kN$$

Longitudinal

$$V_c = \frac{M_o}{H_s} = \frac{28080}{21} = 1337kN$$

Different shear mechanisms of concrete without shear reinforcement

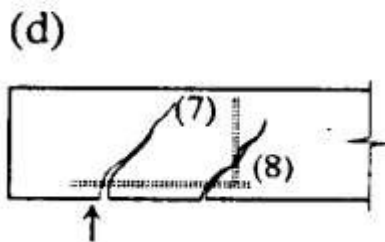
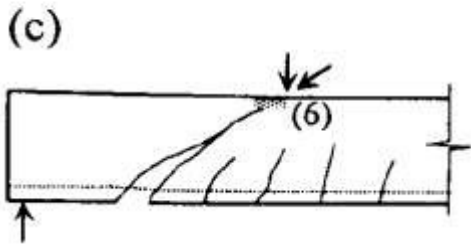
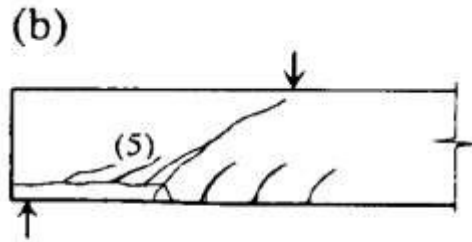
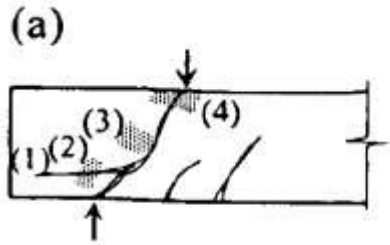


Concrete shear mechanisms:

- friction
- dowel
- compression zone

$$\begin{aligned}
 V &= \frac{dM}{dx} = \frac{d(F_s z)}{dx} \\
 &= \frac{dF_s}{dx} z + F_s \frac{dz}{dx}
 \end{aligned}$$

Shear failure modes

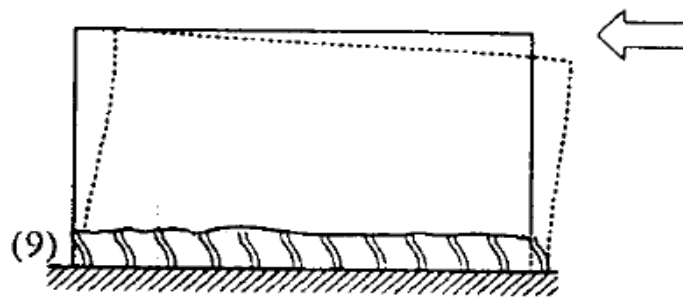
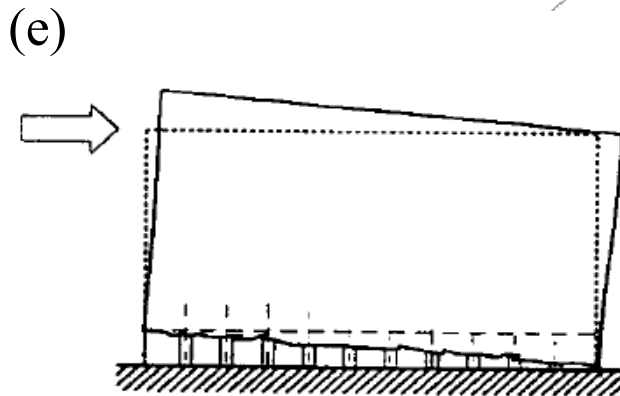


(a) Arch failure

(b) Compression diagonal failure

(c) Exceeded principal tension stresses

(d) Failure of the truss mechanism



(e) Sliding failure – short piers

First the shear strength of the concrete without shear reinforcement is checked

Shear strength of concrete in pier S14

$$V_{Rdc} = \left[C_{Rd,c} k (100 \rho f_{ck})^{1/3} + k_1 \sigma_{cp} \right] b_w d$$

$$C_{Rd,c} = 0,18/1,5 = 0,12$$

$$k_1 = 0,15$$

$$N_{Ed} = 14329 \text{ kN}$$

$$A_c = 3,76$$

$$f_{ck} = 25 \text{ MPa}$$

$$\sigma_{cp} = \frac{N_{Ed}}{A_c} = \frac{14329}{3,76 \cdot 1000} = 3,81 \text{ MPa} > 0,2 f_{cd} = 0,2 \frac{25}{1,5} = 3,3 \text{ MPa}$$

$$\sigma_{cp} = 3,3 \text{ MPa}$$

Transverse direction

$$\gamma_{Bd} = 1,25 + 1 - \frac{3,5 \cdot 3256}{5448} = 0,16 \Rightarrow \gamma_{Bd} = 1,0$$

$$d = (350 - 4 - 1,2)0,9 \cdot 10 = 3103 \text{ mm}$$

$$b_w = (40 - 4 - 1,2)10 \cdot 2 = 696 \text{ mm}$$

$$k = 1 + \sqrt{\frac{200}{d}} = 1 + \sqrt{\frac{200}{3103}} = 1,25 < 2$$

$$\rho = \frac{148 \cdot 3,81}{310,3 \cdot 69,6} = 0,026 > 0,02 \quad \rho = 0,02$$

$$V_{Rd,c}^* = \left[0,12 \cdot 1,25 (100 \cdot 0,02 \cdot 25)^{1/3} + 0,15 \cdot 3,33 \right] 3103 \cdot 696 / 1000 = 2272 \text{ kN} < 5448 \text{ kN}$$

$$V_{Rd,max} = 0,5 b_w d v f_{cd} = 0,5 \cdot 0,696 \cdot 3,103 \cdot 0,54 \cdot 16,7 \cdot 1000 = 9738 \text{ kN} > 5448 \text{ kN}$$

Longitudinal direction

$$v = 0,6 \left(1 - \frac{f_{ck}}{250} \right) = 0,6 \left(1 - \frac{25}{250} \right) = 0,54$$

$$d = (200 - 4 - 1,2)0,9 \cdot 10 = 1753 \text{ mm}$$

$$b_w = (40 - 4 - 1,2)10 \cdot 2 = 696 \text{ mm}$$

$$k = 1 + \sqrt{\frac{200}{d}} = 1 + \sqrt{\frac{200}{1753}} = 1,34 < 2$$

$$\rho = \frac{148 \cdot 3,81}{175,3 \cdot 69,6} = 0,046 > 0,02 \quad \rho = 0,02$$

$$\gamma_{Bd} = 1,25 + 1 - \frac{3,5 \cdot 2261}{3230} = -0,20 \Rightarrow \gamma_{Bd} = 1,0$$

$$V_{Rd,c}^* = \left[0,12 \cdot 1,34 (100 \cdot 0,02 \cdot 25)^{1/3} + 0,15 \cdot 3,33 \right] 1753 \cdot 696 / 1000 = 1332 \text{ kN} < 3230 \text{ kN}$$

Total shear force should be sustained by the shear reinforcement

Transverse direction

$$V_{wd} = \frac{A_{sw}}{s} z f_{ywd} = \frac{1,13 \cdot 4}{10} 0,9 \cdot 310,3 \cdot \frac{50}{1,15} = 5488kN = 5488kN$$

Longitudinal direction

$$V_{wd} = \frac{A_{sw}}{s} z f_{ywd} = \frac{1,13 \cdot 4}{10} 0,9 \cdot 175,3 \cdot \frac{50}{1,15} = 3100kN \sim 3230kN(96\%)$$

4 legs stirrups $\phi 12/10\text{cm}$

Shear strength of concrete in pier S21

Transverse direction

$$d = (350 - 4 - 0,8)0,9 \cdot 10 = 3107 \text{ mm}$$

$$b_w = (40 - 4 - 0,8)10 \cdot 2 = 704 \text{ mm}$$

$$k = 1 + \sqrt{\frac{200}{d}} = 1 + \sqrt{\frac{200}{3107}} = 1,25 < 2$$

$$\gamma_{Bd} = 1,25 + 1 - \frac{3,5 \cdot 1408}{2237} = 0,05 \Rightarrow \gamma_{Bd} = 1,0$$

$$\rho = \frac{94 \cdot 1,54}{310,7 \cdot 70,4} = 0,0066 < 0,02$$

$$V_{Rd,c}^* = \left[0,12 \cdot 1,25 (100 \cdot 0,0066 \cdot 25)^{1/3} + 0,15 \cdot 3,33 \right] 3107 \cdot 704 / 1000 = 1931 \text{ kN} < 2237 \text{ kN}$$

Longitudinal direction

$$d = (200 - 4 - 0,8)0,9 \cdot 10 = 1757 \text{ mm}$$

$$b_w = (40 - 4 - 0,8)10 \cdot 2 = 704 \text{ mm}$$

$$k = 1 + \sqrt{\frac{200}{d}} = 1 + \sqrt{\frac{200}{1757}} = 1,34 < 2$$

$$\gamma_{Bd} = 1,25 + 1 - \frac{3,5 \cdot 688}{1377} = 0,50 \Rightarrow \gamma_{Bd} = 1,0$$

$$\rho = \frac{86 \cdot 1,54}{175,7 \cdot 70,4} = 0,0107 < 0,02$$

$$V_{Rd,c}^* = \left[0,12 \cdot 1,34 (100 \cdot 0,0107 \cdot 25)^{1/3} + 0,15 \cdot 3,33 \right] 1757 \cdot 704 / 1000 = 1211 \text{ kN} < 1337 \text{ kN}$$

Total shear force is sustained by the shear reinforcement

Transverse direction

$$V_{wd} = \frac{A_{sw}}{s} z f_{ywd} = \frac{0,5 \cdot 4}{10} 0,9 \cdot 310,7 \cdot \frac{50}{1,15} = 2431 kN > 2237 kN$$

Longitudinal direction

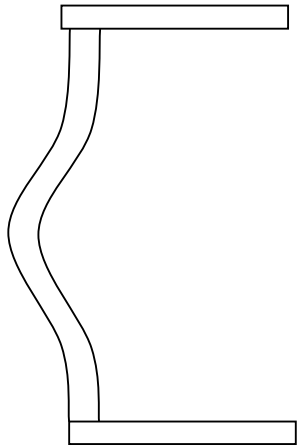
$$V_{wd} = \frac{A_{sw}}{s} z f_{ywd} = \frac{0,5 \cdot 4}{10} 0,9 \cdot 175,7 \cdot \frac{50}{1,15} = 1375 kN > 1337 kN$$

4 leg stirrups $\phi 8/10\text{cm}$

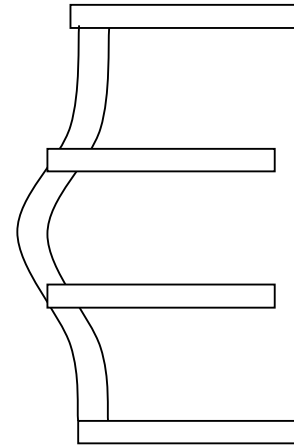
Buckling of the longitudinal reinforcement

Two possible failure modes

Large distance between stirrups



Small amount of the transverse reinforcement



Maximum distance between stirrups

Pier S14

$$s_L = \delta \phi$$

$$\delta = 2,5(ftk / fyk) + 2,25 = 2,5 \cdot 1,35 + 2,25 = 5,625$$

$$s_L = 5,625 \cdot 2,2 = 12,4 \text{ cm}$$

ftk – tensile strength of the transverse reinforcement
fyk – yield strength of the transverse reinforcement

Pier S21

$$s_L = \delta \phi$$

$$\delta = 2,5(ftk / fyk) + 2,25 = 2,5 \cdot 1,35 + 2,25 = 5,625$$

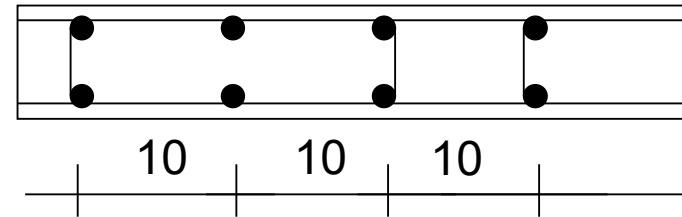
$$s_L = 5,625 \cdot 1,4 = 7,9 \text{ cm}$$

Maximum distance between stirrups legs (the procedure was incorrect, later it was changed)

Pier S14

$$\min\left(\frac{A_t}{s_l}\right) = \frac{\sum A_s \cdot f_{st}}{1,6 f_{yt}} = \frac{3\,381}{1,6} = 714 \text{ mm}^2 / \text{m}$$

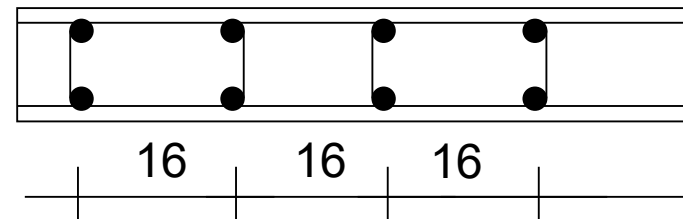
$$s_l \leq \frac{A_t}{714} = \frac{113}{714} = 0,158 \text{ m} = 15,8 \text{ cm}$$



Pier S21

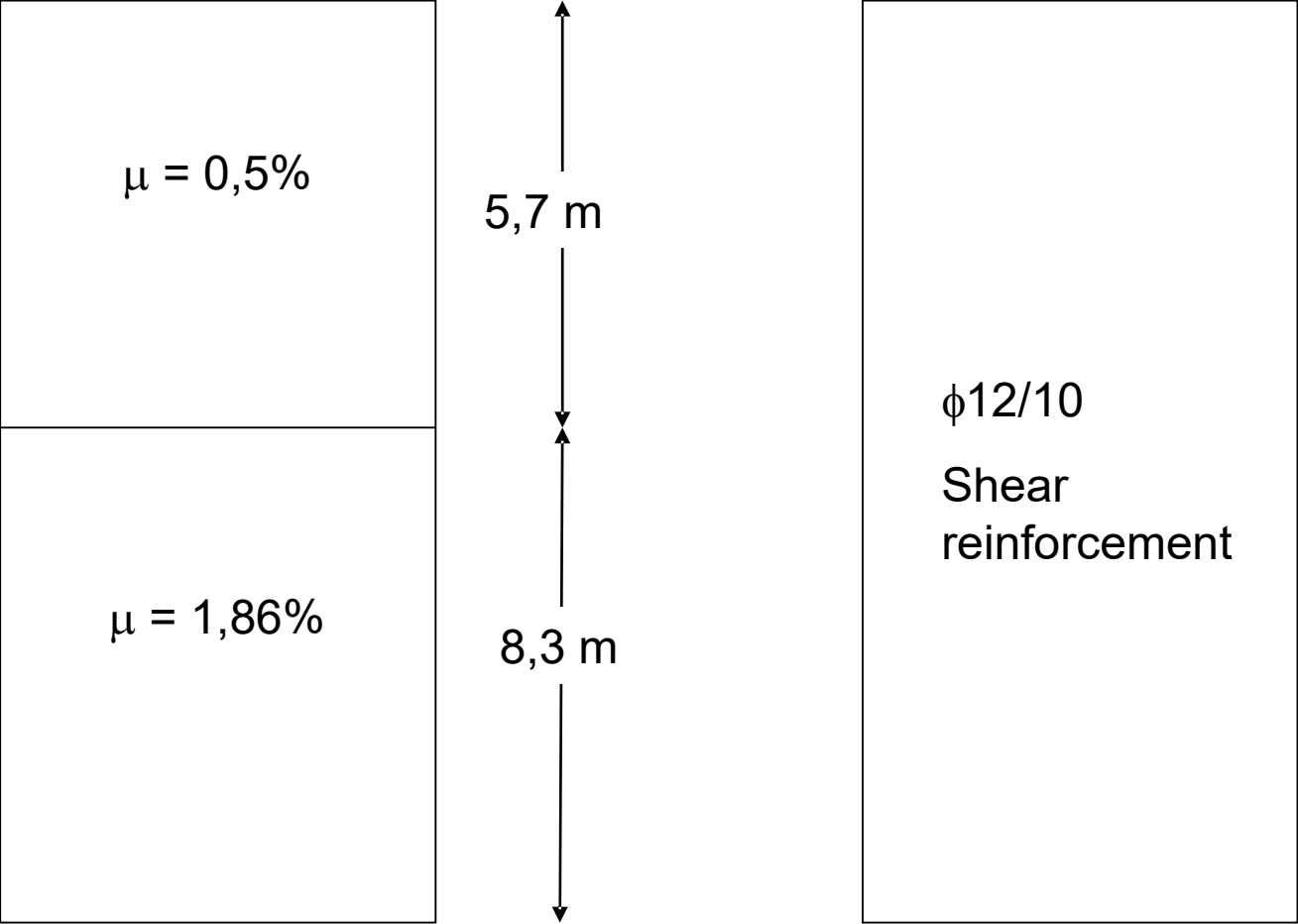
$$\min\left(\frac{A_t}{s_l}\right) = \frac{\sum A_s \cdot f_{st}}{1,6 f_{yt}} = \frac{2 \cdot 154}{1,6} = 192 \text{ mm}^2 / \text{m}$$

$$s_l \leq \frac{A_t}{192} = \frac{50}{192} = 0,260 \text{ m} = 26 \text{ cm}$$



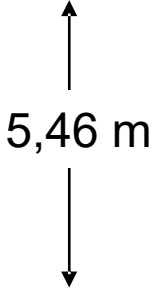
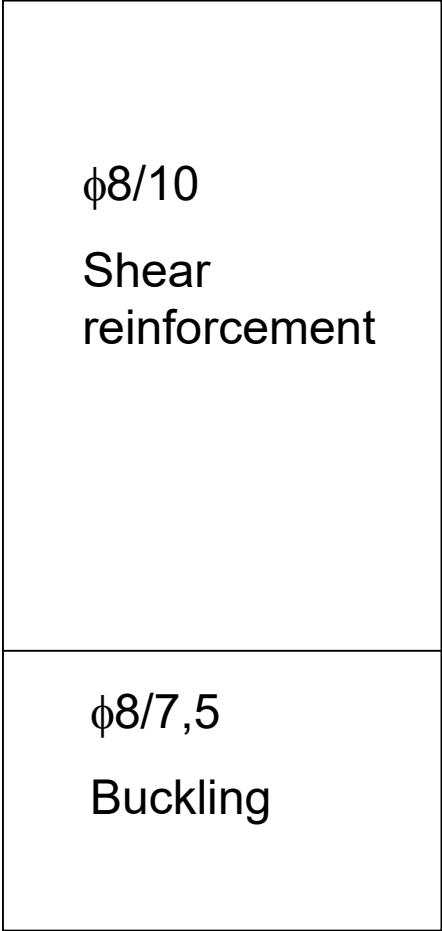
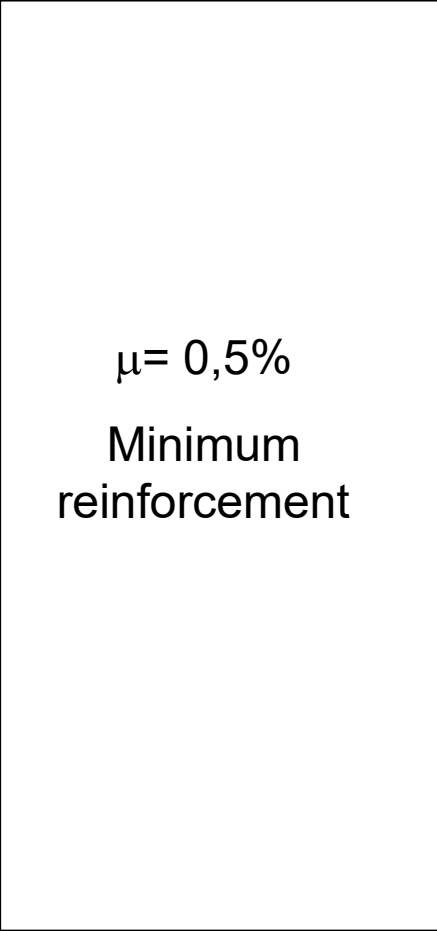
Summary of the reinforcement

Pier S14



Summary of the reinforcement

Pier S21



Capacity design:

Superstructure

Bearings

Foundations

Abutments